國立中山大學 107 學年度碩士暨碩士專班招生考試試題

科目名稱：離散數學【資工系碩士班甲組】　　題號：434004

※本科目依簡章規定「不可以」使用計算機(問答申論題)　　

There are 6 problems in this test. Note that you should write down detailed steps for the solution to each problem; otherwise, no credits for that problem will be given.

1. Find the coefficient of x^7 in each of the following formulas.
   (a) [10%] $((x^2-3x)/(1-x)^5)+3x^7+5$.
   (b) [10%] $-2/((x-1)(x-2))$.

2. Consider the counting of the number of onto functions $H: A \to B$ where $|A| \geq |B|$.
   (a) [10%] Find the exponential generating function for the above counting such that the coefficient of $\frac{x^{|A|+2}}{|A|!}$ is the answer of the above counting.
   (b) [10%] Apply the above exponential generating function to find the answer of the above counting where $|A| = 10$ and $|B| = 3$.

3. Let $\Sigma = \{0, 1\}$ be an alphabet and $A = \{1, 00, 10\}$ be a subset of Σ^*.
   (a) [10%] For each integer $n \geq 1$, let a_n be the number of strings in A^* of length n. Find and solve a recurrence relation for a_n.
   (b) [10%] For each integer $n \geq 1$, let b_n be the number of strings in A^* which are of length n and exactly divided by 2 when we regard each of the strings as a binary number. Find b_n.

4. Consider the additive group $(\mathbf{Z}_8, +)$.
   (a) [5%] What is the order of 6? Why?
   (b) [5%] Find all generators of the group.

5. Consider the multiplicative group $(\mathbf{Z}^*_{196}, \cdot)$.
   (a) [5%] What is the order of the group? Why?
   (b) [5%] Find the inverse of 25.

6. Let $(\mathbf{Z}\times\mathbf{Z}, \oplus)$ be the group with $(a, b)\oplus(c, d) = (a+c+2, b+d-2)$ for any $(a, b), (c, d) \in \mathbf{Z}\times\mathbf{Z}$ where $a+c+2$ and $b+d-2$ are computed using ordinary addition and subtraction in $\mathbf{Z}$.
   (a) [2%] $3(4, 5) = (4, 5)\oplus(4, 5)\oplus(4, 5) = ?$
   (b) [5%] What is the identity of the group?
   (c) [5%] What is the inverse of (a, b)?
   (d) [8%] Let $(\mathbf{G}, +)$ be an additive group and let $h: \mathbf{Z}\times\mathbf{Z} \to \mathbf{G}$ be a group homomorphism where $h(3, 2) = u$ and $h(-2, 6) = v$. Please express $h(18, 22)$ in terms of u and v.

試題隨卷繳回