國立中山大學 107 學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學【機電系碩士班乙組、丙組】　　題號：438001

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

1. (15%) Solve the IVP: $y'' + 6y' + 9y = e^{-x}\cos 2x,\ y(0) = 1, y'(0) = -1.$
   (a) Find the general solution $y_h = c_1y_1 + c_2y_2$. (5%)
   (b) Show the independence of y_1 and y_2. (5%)
   (c) Find the particular solution. (5%)

2. (25 %) Each of the two tanks contains 200 gal of water, where initially 100 lb (tank T_1) and 200 lb (tank T_2) of salt are dissolved. The inflow, circulation, and outflow are shown in **Fig. 1**. The mixture is kept uniform by stirring.

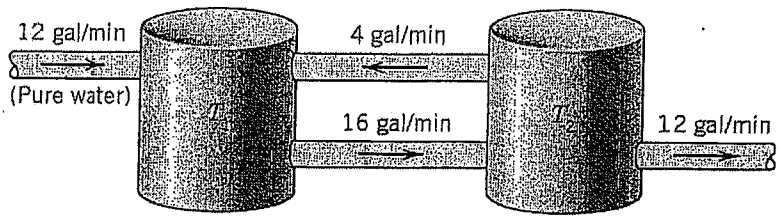

Fig. 1

   (a) Set up the model. (5%)
   (b) Solve the system. (10%)
   (c) Type of critical point and its stability. (5%)
   (d) What will happen if tends to infinity? (5%)

3. (10%) Find the Inverse Laplace Transform of $\frac{4(e^{-2s}-2e^{-5s})}{s^2+4}$.

4. (8%) Find the volume of a parallelepiped, as shown in **Fig. 2**, determined by the three edge vectors: $\mathbf{a} = [2, 1, 5],\ \mathbf{b} = [1, 4, 3],\ \mathbf{c} = [4, 5, 1]$.

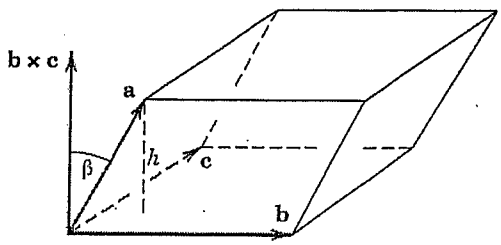

Fig. 2

5. (8%) Find the tangential acceleration and normal acceleration of the motion given by $\mathbf{r}(t) = \left[\sin t,\ \cos t,\ \frac{1}{2}t^2\right]$ (t = time).

6. (10%) Evaluate the surface integral $\iint_S \boldsymbol{F}\cdot\boldsymbol{n}\,dA$, where $\boldsymbol{F} = [xy, yz, zx]$ and S is the surface of the cone $x^2 + y^2 \le 4z^2,\ 0 \le z \le 3$.

7. (10%) Evaluate the surface integral $\iint_S (\text{curl}\,\boldsymbol{F})\cdot\boldsymbol{n}\,dA$, where $\boldsymbol{F} = [y^3, -x^3, 0]$, S: $x^2 + y^2 \le 1,\ z = 0$.

8. (14%) Solve the partial differential equation: $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with the boundary conditions: $\frac{\partial u}{\partial x}(0, t) = 0$ and $\frac{\partial u}{\partial x}(L, t) = 0$ (for all $t \ge 0$) and the initial condition: $u(x, 0) = \cos\frac{2\pi}{L}x$.