國立中山大學107學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學甲【電機系碩士班甲組、己組、電波領域選考】　　題號：431002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）　　

第1題到第10題為單選題，每題四分，共計四十分。每題請選出一個最正確選項，答錯倒扣一分。第1題到第10題中，若$z := x + jy$是一個複數，則 x, y 是實數而 j 代表 $\sqrt{-1}$。

1. Given a continuous-time periodic signal

$$f(t) = 4 + 8\cos\left(\frac{2\pi}{3}t\right) - \sin\left(\frac{5\pi}{3}t\right).$$

Let a_k and ω_0 be the coefficients and fundamental frequency of the Fourier series of $f(t)$ respectively. Then which of the following statements is correct?

(A) $a_{-1} + a_5 = -j/2$, $\omega_0 a_0 = 4\pi/3$, $a_0 a_{-1} = 0$.

(B) $a_{-1} + a_2 = 4$, $\omega_0 a_{-5} = -j\pi/6$, $a_0 a_2 = 16$.

(C) $a_{-2} + a_5 = 4 + j/2$, $\omega_0 a_2 = 4\pi/3$, $a_1 a_5 = j/2$.

(D) $a_0 < 5$, $\omega_0 < 2$, $a_5 > 0$.

(E) None of the above statements are correct.

2. Consider a discrete signal $x(n) = \cos(2n\pi/N)$, where integer N is the fundamental period. Let a_k be the coefficients of the discrete-time Fourier series of $x(n)$. Then which of the following statements is correct?

(a) $a_1 = -1/2$, $a_2 = 1/2$.

(b) $a_1 = a_2 = j/2$

(c) $a_{N+1} = 1/2$, $a_{N-1} = -1/2$.

(d) $a_{N+1} = a_{N-1} = 1/2$

(e) None of the above statements are correct.

3. Consider an LTI system whose impulse response is $G(j\omega) = 1/(a + j\omega)$, $a > 0$. Suppose that there is an input signal $X(j\omega) = 1/(a + j\omega)$. Assume that the output signal is $y(t) = \beta e^{\gamma t} u(t)$, where $u(t)$ is a unit-step signal. Then which of the following statements is correct?

(a) $\beta + \gamma = -a$

(b) $\beta\gamma = -a$

(c) $\beta - \gamma = t + a$

(d) $\beta\gamma = at$

(e) None of the above statements are correct.

4. Consider the following three systems, where $x[n]$ or $x(t)$ is the system input, $y[n]$ or $y(t)$ denotes the system output, and

I. $y[n] = x[-n]$, $-\infty < n < \infty$

II. $y(t) = [\sin(2t)]x(t)$

III. $y[n] = \begin{cases} x[n-4], & n \geq 1 \\ 0, & n = 0 \\ x[n-1], & n \leq -1 \end{cases}$

Which of the following statements is correct?

(a) I is time-invariant, II is linear, III is causal.

(b) I is memoryless, II is causal, III is nonlinear.

(c) I is stable, II is linear, III is memoryless.

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(d) I is causal, II is memoryless, III is linear.
(e) None of the above statements are correct.

5. Let $z=1-j\sqrt{3}$. Assume that $z^{1/5}=re^{j\theta}$, where θ denotes the principal argument. Then which of the following statements is correct?
(a) $r=2,\ \theta=\pi/5$
(b) $r=4^{1/5},\ \theta=-\pi/15$
(c) $r=2^{1/5},\ \theta=\pi/15$
(d) $r=2^{1/5},\ \theta=-\pi/5$
(e) None of the above statements are correct.

6. Which one of the following functions, where $z=x+jy$ is a complex variable, is analytic?
(a) $f(z)=\overline{z}^2$, where $\overline{z}=x-jy$.
(b) $f(z)=xy+jx$
(c) $f(z)=x^2-jy^2$
(d) $f(z)=x^2+y^2+j2xy$
(e) None of the above statements are correct.

7. Let z be a complex number. Which of the following statements is correct?
(a) $Log(z_1z_2)=Log(z_1)+Log(z_2)$, where $Log(z)$ is the principal value of the complex logarithm.
(b) $\cos(j)$ is not a real value.
(c) $j^{j2}=e^{-j(4n+1)\pi}$, $n=0,\ \pm1,\ \pm2,\ldots\ldots$
(d) $\overline{z^2+2z-e^z}=\overline{z}^2+2\overline{z}-\exp(\overline{z})$, where $\exp(z)=e^z$.
(e) None of the above statements are correct.

8. Let $f(z)=z/(z^2+9)$, and C be a circle $|z-j2|=4$ in counterclockwise direction. The evaluation of $\oint_C f(z)dz$ is $\alpha+j\beta$. Then which of the following statements is correct?
(a) $\alpha<0,\ \beta>0$
(b) $\alpha>0,\ \beta>0$
(c) $\alpha=0,\ 1<\beta<4$
(d) $\alpha>0,\ -3<\beta<4$
(e) None of the above statements are correct.

9. Let $f(z)=\overline{z}$, and C be the right-hand half of the circle $|z|=2$ form $z=j2$ to $z=-j2$. Compute the value of $\int_C f(z)dz=\alpha+j\beta$. Then which of the following statements is correct?
(a) $\alpha=0,\ -15<\beta<0$
(b) $a>0,\ 0<\beta<15$
(c) $-12<\alpha+\beta<12$
(d) $-2<\alpha\beta<-12$
(e) None of the above statements are correct

10. The Laurent series of $e^{1/z}$ is

$$e^{1/z}=1+\frac{1}{z}+\frac{1}{\alpha z^2}+\frac{1}{\beta z^3}+\ldots\ldots,\ 0<|z|<\infty$$

Which of the following statements is correct?

(a) $\alpha\beta = 1/6$

(b) $\alpha\beta = 1/12$

(c) $\alpha + \beta = 5/6$

(d) $\alpha + \beta = 1/3$

(e) None of the above statements are correct.

以下第11題到第13題中之所有的提問，都不需要寫出推導過程，只要寫出答案即可，答案正確就得分。

11. (10%) Let $A \in \mathbb{R}^{m\times n}$ and $\mathbf{b} \in \mathbb{R}^m$.

(a) (5%) Suppose that $m = n = 3$, $A = \begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & 4 \\ -1 & -4 & 2 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$. Find the set of all solutions to the equation $A\mathbf{x} = \mathbf{b}$ if it is consistent. Otherwise, find vector $\mathbf{p}$ to solve $\min_{\mathbf{p}\in R(A)} \|\mathbf{b}-\mathbf{p}\|_2$ and, moreover, compute the value of $\min_{\mathbf{p}\in R(A)} \|\mathbf{b}-\mathbf{p}\|_2$.

(b) (5%) When the equation $A\mathbf{x} = \mathbf{b}$ is unsolvable, we may consider the so-called least squares problem to find a set of solutions, having the least squares error, from solving a normal equation. Suppose that $rank(A) = k < \min(m, n)$ and let $A = BC$ be a full rank decomposition of A. Use the known matrices B, C, and $\mathbf{b}$ to describe the unique projection vector $\mathbf{p}$ of $\mathbf{b}$ onto $R(A)$ with the least $\|\mathbf{b}-\mathbf{p}\|_2$.

12. (10%) Let $f_1 = x+\alpha$ and $f_2 = x-\alpha$, $\alpha \in \mathbb{R}$, be two vectors in the vector space $C[0, 1]$ with inner product $\langle f, g\rangle := \int_0^1 f(x)g(x)dx$.

(a) (4%) Denote the angle between f_1 and f_2 by θ. Find all possible values of α^2 such that $\theta = \pi/4$.

(b) (6%) Now set $\alpha = 1$. Find functions g_1 and g_2 such that $\{g_1, g_2\}$ is an orthonormal set that satisfies $Span(g_1) = Span(f_1)$ and $Span(g_1, g_2) = Span(f_1, f_2)$.

13. (10%) Consider a linear transformation $L: P_2 \to \mathbb{R}^2$ defined by $L(p(x)) := \begin{bmatrix} \int_0^1 p(x)dx \\ \beta\cdot p(0) + \int_1^\beta p(x)dx \end{bmatrix}$, for every $p(x) \in P_2$, with $\beta > 1$.

(a) (4%) Find all possible values of β such that L^{-1}, the inverse of L, does not exist.

(b) (6%) Suppose that L^{-1} exists. Find the matrix representation of L^{-1} corresponding to the ordered bases $\{x+1, x-1\}$ and $\left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}\right\}$ for P_2 and $\mathbb{R}^2$, respectively.

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以下第 14 題到第 15 題中之所有的提問，需要寫出推導過程或詳細說明理由，答案正確但沒有推導過程或說明不正確，將酌扣分數或不給分。

14. (20%) Consider the following set of differential equations

$$\ddot{x}_1(t) = -2x_1(t) + 2x_2(t)$$
$$\ddot{x}_2(t) = 2x_1(t) + 5x_2(t) + u(t)$$

(a) (15%) Let $u(t) \equiv 0$ and the initial conditions be $x_1(0) = x_2(0) = 1$, $\dot{x}_1(0) = \dot{x}_2(0) = 0$. Find the solutions of the differential equations.

(b) (5%) Let initial conditions be $x_1(0) = x_2(0) = \dot{x}_1(0) = \dot{x}_2(0) = 0$, and $u(t)$ be the unit step function. Does the solutions of the differential equations converge to constant values as time approaches infinity? Justify your answers.

15. (10%) Evaluate the following integral

$$\int_0^{\infty} \int_{x^2}^{\infty} x\mathrm{e}^{-y^2} dy dx$$

End of Examination