國立中山大學 107 學年度碩士暨碩士專班招生考試試題

科目名稱：高等微積分【應數系碩士班丙組】　　　　題號：424004

※本科目依簡章規定「不可以」使用計算機(問答申論題)　　　　共1頁第1頁

1. [10%] Find the Taylor series expansion for the function

$$f(x) = \frac{1}{1+2x^2}$$

about $x = 0$ and find the convergence of interval of the series.

2. [15%] Define $f : \mathbb{R}^2 \to \mathbb{R}$ as

$$f(x,y) = \frac{y^{5/2}}{x^2+y^2}$$

for $(x,y) \neq (0,0)$ and $f(0,0) = 0$. Is f continuous at $(0,0)$? Verify your assertion.

3. [15%] Let $\{a_n\}$ be a positive sequence with $\sum a_n$ divergent. Show that the series

$$\sum \frac{a_n}{1+a_n}$$

also diverges.

4. [15%] Let $\{f_n\}$ be a sequence of continuous functions defined on $[0,1]$, and suppose that the limit $\lim_{n\to\infty} f_n(x) = f(x)$ exists for any $x \in [0,1]$.
(1)[7%] Is f continuous on $[0,1]$? Verify your assertion.
(2)[8%] Is it true that

$$\lim_{n\to\infty} \int_0^1 f_n(x)\, dx = \int_0^1 f(x)\, dx?$$

Verify your assertion.

5. [15%] Show that the equation

$$x^2 + x + y + \sin(x^2+y^2) = 0$$

determines a unique solution y as a function x near the point $(0,0)$ and show that this unique solution is differentiable at 0. Find the derivative $y'(0)$.

6. [15%] Show that for any continuous function $f : [0,1] \to [0,1]$, there exists a point $\xi \in [0,1]$ for which $f(\xi) = \xi$.

7. (1)[8%] Is the intersection

$$\bigcap_{k=1}^{n} V_k$$

of open sets $V_1, \ldots, V_n$ in some metric space X open in X? Verify your assertion.
(2)[7%] Is your assertion in (1) still true if the finite intersection is replaced with a countable intersection of open sets in X? Verify your assertion.

試題隨卷繳回