國立中山大學107學年度碩士暨碩士專班招生考試試題

科目名稱：線性代數【應數系碩士班丙組】　　　　題號：424003

※本科目依簡章規定「不可以」使用計算機(問答申論題)　　　　共1頁第1頁

請詳列計算証明之過程. 作答時請標明題號. 共七題.

1. Find the general solution of $\begin{cases} x_1 - x_2 + x_3 + 2x_4 = 2 \\ x_1 + x_2 - 2x_3 - x_4 = 1 \end{cases}$. (10%)

2. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & 0 \\ 2 & 0 & 1 \end{bmatrix}$. Is A invertible? Find A^{-1} if it exists. (10%)

3. Let $A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 2 & -1 & -1 \\ 0 & 1 & -1 & -1 \end{bmatrix}$. Find a basis for the null space of A and a basis for the image space of A. (15%).

4. Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. Find an orthogonal P such that $P^{-1}AP$ is diagonal. Also, find a matrix B such that $B^{107} = A$. (15%)

5. Let A be an $n \times n$ matrix and $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_k\}$ be a subset of $\mathbf{R}^n$.
   (a) Prove or disprove: If A is invertible and $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_k\}$ is independent, then $\{A\mathbf{x}_1, A\mathbf{x}_2, \ldots, A\mathbf{x}_k\}$ is independent. (10%)
   (b) Prove or disprove: If $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_k\}$ and $\{A\mathbf{x}_1, A\mathbf{x}_2, \ldots, A\mathbf{x}_k\}$ are independent, then A is invertible. (10%)

6. Define $T : \mathbf{R}^2 \to \mathbf{R}^2$ by $T(x, y) = (x - 2y, x + y)$. Find the matrix of T relative to the ordered basis $B = \{(1,1)^T, (-1,1)^T\}$. (15%)

7. Let $A = \begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix}$. Find $A^{14} + A^{13} - 5A^{12} + A^5 + A^4 - 5A^3 + A^2 - A + 2I$. (15%)

試題隨卷繳回