國立中山大學107學年度碩士暨碩士專班招生考試試題

科目名稱：線性代數【應數系碩士班乙組】 題號：424005

※本科目依簡章規定「不可以」使用計算機(問答申論題) 共1頁第1頁

計算題：共6題，子題分數平均分配。答題時，每題都必須寫下題號與詳細步驟。

[1]. (18%) Answer the following questions.

(a) What is the vector space?

(b) What is the linear transformation?

(c) What is the rank of a matrix?

[2]. (14%) Let

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ -2 & -1 & 2 & -2 \\ 0 & 0 & 1 & 3 \\ 3 & 4 & 2 & 5 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}.$$

(a) Solve the linear system $A\mathbf{x} = \mathbf{b}$ for $\mathbf{x}$.

(b) Compute $\det(A)$.

[3]. (18%) Let

$$A = \begin{bmatrix} 2 & -6 & 3 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & -2 & 2 & 0 \\ 0 & -6 & 3 & 3 \end{bmatrix}.$$

(a) Find the characteristic polynomial.

(b) Find the eigenvalues and the corresponding eigenvectors.

(c) Find a matrix C and a diagonal matrix D such that $D = C^{-1}AC$.

[4]. (16%) Let

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 6 \\ 0 \\ 3 \end{bmatrix}.$$

(a) Use the Gram-Schmidt process to find an orthonormal basis for the column space of A.

(b) Solve the least squares problem $A\mathbf{x} = \mathbf{b}$.

[5]. (16%) Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be defined by

$$T(\mathbf{x}) = \begin{bmatrix} x_1 + x_2 + 3x_3 \\ x_1 - x_2 + 2x_3 \\ 3x_1 + 2x_2 \end{bmatrix}.$$

(a) Find a matrix A such that $T(\mathbf{x}) = A\mathbf{x}$ for each $\mathbf{x} = (x_1, x_2, x_3)^T$ in $\mathbb{R}^3$.

(b) Let $\beta = ([1,1,1], [1,1,0], [1,0,0])$ be an ordered basis of $\mathbb{R}^3$.

Find the matrix representation $B = [T]_\beta$ of T with respect to β.

[6]. (18%) Let A be an $m \times n$ matrix. Show that

(a) The nullspace of A^TA is the nullspace of A.

(b) A^TA and A have the same rank.

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試題隨卷繳回