

國立中山大學 107 學年度碩士暨碩士專班招生考試試題

科目名稱：機率與統計【應數系碩士班甲組】

題號：424006

※本科目依簡章規定「不可以」使用計算機(問答申論題)

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答題時，每題須寫下題號與詳細步驟。請依題號順序作答，不會作答題目請寫下題號並留空白。

Notation:

i.i.d.: identically independently distributed; pdf: probability density function; MLE: maximum likelihood estimator; $\exp(\theta)$ random variable means a random variable with exponential distribution with a parameter θ and its pdf is $f(x|\theta) = \frac{1}{\theta}e^{-\frac{x}{\theta}}, x > 0$. \bar{X} is the sample mean of X_1, \dots, X_n . $\text{Bin}(n, p)$ indicates the Binomial distribution with n independent Bernoulli trial and each trial has success rate p . $\text{Unif}[a, b]$ represents the uniform distribution within $[a, b]$.

- (15%) The joint moment generating function for random variables U and V is defined as $M(s, t) = E(\exp(sU + tV))$. X and Y are independent random variables with common moment generating function $M(t) = \exp(5t^2)$. Let $U = X + Y + 3$ and $V = 2X - 2Y$. What is the joint moment generating function for U and V ?
- (15%) Assume the distribution of N is $\text{Bin}(m, p)$. Conditional on $N = n$, the distribution of Y is $\text{Bin}(n, q)$. What is the unconditional distribution of Y ?
- (15%) Let X_1, \dots, X_{2n} be iid $\text{Unif}[0, 3]$. The order statistics are $X_{(1)} < X_{(2)} < \dots < X_{(2n)}$. What is the expectation of $X_{(n)}$?
- (15%) Let $Y_i \sim \text{Bin}(n_i, p_i), i = 1, \dots, m$, be mutually independent. Please derive the likelihood ratio test with significance level α for the null hypothesis

$$H_0 : p_1 = \dots = p_m$$

against the alternative hypothesis that not all the p_i are equal. You have to specify the test statistic and the asymptotic rejection region.

- (20%) X_1, \dots, X_n are i.i.d $\exp(\theta)$. Please answer the following questions.
 - (5%) Prove \bar{X} and $\frac{X_1}{\bar{X}}$ are independent.
 - (15%) Use (a) to derive the UMVUE for the parameter $P(X_1 > t) = e^{-t/\theta}$.
- (20%) X_1, \dots, X_n are independent random variables with X_i being distributed with $N(\mu, w_i\sigma^2)$, where w_i are known constants and μ and σ^2 are unknown parameters).
 - (10%) Please find the MLE for μ .
 - (10%) Calculate and the mean squared errors of the MLE derived from (a) and \bar{X} for μ . Which one has smaller mean squared error (you have to prove your answer)?

試題隨卷繳回