

國立中山大學 107 學年度碩士暨碩士專班招生考試試題

科目名稱：基礎數學【應數系碩士班甲組】

題號：424001

※本科目依簡章規定「不可以」使用計算機(問答申論題)

共 1 頁第 1 頁

共十題，每題 10 分。答題時，每題都必須寫下題號與詳細步驟。
請依題號順序作答，不會作答題目請寫下題號並留空白。

1. Use implicit differentiation to find the tangent line at $(2, 2)$ of the graph of the function $2x^3 - 3y^2 = 4$. Also find the second derivative $\frac{d^2y}{dx^2}$ at $(2, 2)$.

2. Evaluate the limits.

(a) $\lim_{x \rightarrow \frac{\pi}{4}} \tan 2x \cdot \tan(\frac{\pi}{4} - x)$.

(b) Suppose $\lim_{x \rightarrow \infty} f'(x) = A$, $a > 0$, find $\lim_{x \rightarrow \infty} \{f(x+a) - f(x)\}$.

3. Consider a segment of the curve described by the equation $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$ in the first quadrant (i.e., when $0 \leq x \leq 1$ and $0 \leq y \leq 1$).

(a) Find the length of the curve.

(b) Find the area of the surface generated by revolving the curve about the x -axis.

4. Find the antiderivative $\int \frac{e^x + 1}{e^{2x} - e^x + 2} dx$.

5. Consider the power series $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{(n+1) \ln(n+1)}$.

(a) Determine its radius of convergence.

(b) Determine its interval of convergence.

6. Find and classify the critical points of the function $f(x, y) = x^4 + y^4 - 4xy$.

7. Find the area of the sphere $x^2 + y^2 + z^2 = 4$ lying inside the cylinder $(x-1)^2 + y^2 = 1$.

8. Diagonalize the matrix $\begin{bmatrix} 2 & -2 & -2 \\ 3 & -3 & -2 \\ 2 & -2 & -2 \end{bmatrix}$ with the real eigenvalues $\lambda = -2, -1, 0$.

9. The given set

$$\left\{ \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 8 \\ 5 \\ -6 \end{bmatrix} \right\}$$

is a basis for a subspace W . Use the Gram-Schmidt process to produce an orthogonal basis for W .

10. Make a change of variable, $x = Py$, that transforms the quadratic form $x_1^2 + 10x_1x_2 + x_2^2$ into a quadratic form with no cross-product term. Give P and the new quadratic form.