

※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Time headway in traffic flow is the elapsed time between two consecutive cars. Let X = time headway between two randomly chosen cars. Consider the probability density function of X as follows. (5 points each)

$$f(x) = \begin{cases} \frac{k}{x^4} & x > 1 \\ 0 & x \leq 1 \end{cases}$$

- (1) Determine the value of k for which $f(x)$ is a legitimate pdf.
 - (2) Obtain the cumulative distribution function.
 - (3) Use the cdf from (2) to determine the probability that headway exceeds 2 sec and also the probability that headway is between 2 and 3 sec.
 - (4) Obtain the mean value of headway and the standard deviation of headway.
2. Test results reported on an experiment in which 120 samples were randomly divided into a control group and a treatment group, each consisting of 60 specimens. The sample mean strength (MPa) and sample standard deviation for the treatment group were 19.9 and 39.1, respectively, whereas these values for the control group were 13.7 and 15.8. (5 points each)
- (1) Calculate a point estimate for the difference between true strength for the treatment and control groups.
 - (2) Do the data provide compelling evidence for concluding that true average strength for the treatment group exceeds that for the control group? Test the appropriate hypotheses at the $\alpha = 0.05$ level.
 - (3) Construct a 95% confidence interval for the overall average strength of the treatment group.

Values Provided for Your Calculations

z	1.04	1.64	1.96	$t_{0.05, 5}$	$t_{0.05, 4}$	$t_{0.025, 4}$	$t_{0.025, 5}$	$F_{0.05, 1, 7}$	$F_{0.05, 1, 8}$	$F_{0.05, 1, 9}$	$F_{0.05, 2, 7}$	$F_{0.05, 2, 8}$	$F_{0.05, 2, 9}$
$\Phi(z)$	0.85	0.95	0.975	2.015	2.132	2.774	2.571	6.61	5.32	5.12	9.55	8.65	8.02

3. The diameter of five steels was measured using two different kinds of calipers. The data are given below. Is there evidence to support that there is a mean difference in the diameter measurements using the two calipers? Using a significance level = 0.05 and show all steps. Make sure to include in your answer the null and alternative hypotheses. For the data, $\bar{d} = 0.007$, and $s_d = 0.00274$. (5 points).

	1	2	3	4	5
Machine 1	0.270	0.275	0.275	0.270	0.280
Machine 2	0.265	0.270	0.270	0.260	0.270

4. Two materials (i.e., X and Y) were tested for their tensile strength for construction work. We would like to explain the behavior of the Y material as a linear function of the X material. A total of 10 pairs of specimens were randomly collected and tested. For the data, $\bar{X} = 75$, $\bar{Y} = 80$; $S_{XX} = \sum(X - \bar{X})^2 = 400$; $S_{YY} = \sum(Y - \bar{Y})^2 = 600$ and $S_{XY} = \sum(X - \bar{X})(Y - \bar{Y}) = 300$. (5 points each)
- (1) State the assumptions of the linear regression model.
 - (2) Estimate the slope and intercept from a simple linear regression analysis of these data.
 - (3) Complete the ANOVA table for this regression model
 - (4) Calculate an estimate of MSE.
 - (5) Using the ANOVA table, make your conclusion. Make sure to include in your answer the null and alternative hypotheses.
 - (6) Calculate the coefficient of determination for the model.
 - (7) Interpret this coefficient of determination in the context of the problem.
5. Find the best answer for each question. (5 points each)
- (1) Suppose that $\bar{x} \sim N(3, 2.5^2)$ and $\bar{y} \sim N(4, 3.5^2)$. Suppose further that \bar{x} and \bar{y} are independent. Find the distribution of the difference $\bar{y} - \bar{x}$. (a) $N(1, 4.3^2)$ (b) $N(1, 2.45^2)$ (c) $N(1, 18.5^2)$ (d) $N(-1, 1^2)$ (e) $N(-1, 6^2)$.
 - (2) What is the confidence level for the interval $\bar{x} \pm 1.64 \cdot \frac{\sigma}{\sqrt{n}}$? (a) 0.80, (b) 0.85, (c) 0.90, (d) 0.95, (e) 0.99.
 - (3) Which of the following is not a valid null hypothesis? (a) $\mu \geq 5$, (b) $\sigma = 0.3$, (c) $\mu_1 - \mu_2 \leq -1$, (d) $\mu < 0.2$, (e) $\sigma_1 / \sigma_2 = 1$.
 - (4) Suppose we created a 95% confidence interval for mean length of time waiting for the bus of (3.12, 8.54) minutes, from our sample mean wait time of 5.83 minutes. Which of the following statements is true? (a) 95% of the time, the true mean μ length of time spent waiting for the bus will fall in the interval I selected above. (b) If I were to repeat the study by keeping track of how long I wait for the bus multiple semesters, about 95% of the time, I would capture the true mean length of time spent waiting for the bus. (c) The probability that the interval I selected above contains 5.83 is 0.95. (d) About 95% of the confidence intervals that I create will contain the mean 5.83 minutes. (e) The probability that the interval I selected above contains the true mean wait time for the bus is 0.95.
 - (5) The following 95% simultaneous confidence intervals (CI) are obtained on the 4 different temperatures in an experiment. Which of the following four statements do you think describes the relationship between μ_1 , μ_2 , μ_3 , and μ_4 ? (a) $\mu_3 = \mu_4$. μ_1 and μ_2 differ from μ_3 and μ_4 . (b) $\mu_1 = \mu_2 = \mu_3$, and μ_4 differs from μ_1 , μ_2 , and μ_3 . (c) $\mu_2 = \mu_4$. μ_2 and μ_4 differ from μ_1 and μ_3 . (d) $\mu_1 = \mu_3 = \mu_4$, and μ_2 differs from μ_1 , μ_3 , and μ_4 . (e) All four μ 's are different from one another.

Difference	$\mu_1 - \mu_2$	$\mu_1 - \mu_3$	$\mu_1 - \mu_4$	$\mu_2 - \mu_3$	$\mu_2 - \mu_4$	$\mu_3 - \mu_4$
CI	(-0.5, -0.1)	(-0.3, -0.2)	(0.2, 0.5)	(0.1, 0.5)	(-0.1, -0.4)	(0.2, 0.4)