編號： 116　　國立成功大學 107 學年度碩士班招生考試試題

系　　所：工程科學系

考試科目：線性代數與機率　　考試日期：0206，節次：3

第1頁，共1頁

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (15%) Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the eigenvalues of an $n \times n$ matrix $\boldsymbol{A}$. Please show that

$$\det(\boldsymbol{A}) = \lambda_1 \cdot \lambda_2 \cdot \cdots \cdot \lambda_n.$$

2. (10%) For a 2×2 matrix $\boldsymbol{A}$, we have $\operatorname{tr}(\boldsymbol{A}) = 7$ and $\det(\boldsymbol{A}) = 12$. Please find the eigenvalues of $\boldsymbol{A}$.

3. (10%) For a 6×8 matrix

$$\boldsymbol{H} = \begin{pmatrix} 0 & 1 & \\ 0 & 2 & \\ 1 & 3 & \\ 2 & 4 & \boldsymbol{I}_6 \\ 3 & 0 & \\ 4 & 0 & \end{pmatrix}$$

where $\boldsymbol{I}_6$ is a 6×6 identity matrix, please find a 2×8 matrix $\boldsymbol{A}$ such that $\boldsymbol{A}\boldsymbol{H}^T = \boldsymbol{0}$.

4. (15%) Please find $|\det(\boldsymbol{A})|$ where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 16 \\ 1 & 8 & 27 & 64 \\ 1 & 16 & 81 & 256 \end{pmatrix}.$$

5. (10%) Let X be a lognormal random variable with parameters μ and σ^2, that is, $\ln X \sim N(\mu, \sigma^2)$. Please find $\mathrm{E}(X^2)$.

6. (15%) For two random variables X and Y, if the covariance $\mathrm{Cov}(X, Y) = 0$, please express the correlation coefficient $\rho(X+Y, X-Y)$ in terms of $\mathrm{Var}(X)$ and $\mathrm{Var}(Y)$.

7. (10%) For two continuous random variables X and Y, please show that $\mathrm{E}[\mathrm{E}(X|Y)] = \mathrm{E}(X)$.

8. (15%) Let $X_1, X_2, \ldots, X_n$ be independent Poisson random variables with parameters $\lambda_1, \lambda_2, \ldots, \lambda_n$, respectively. Also let $X = X_1 + X_2 + \cdots + X_n$.
   a. (5%) Find the moment-generating function of X_1.
   b. (5%) Show that X is a Poisson random variable.
   c. (5%) Find the parameter of the Poisson random variable X.