編號： 79 國立成功大學107學年度碩士班招生考試試題

系　所：機械工程學系

考試科目：工程數學　　考試日期：0205，節次：3

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. For a typical mass-spring-dashpot system, $m\ddot{x} + c\dot{x} + kx = f(t)$,

   where $\ddot{x} = d^2x/dt^2$, $\dot{x} = dx/dt$ and *t* denotes the time variable.

   Assume $f(t) = F_0 \sin(pt)$.

   (a) If $p \neq \sqrt{k/m}$, and $c = 0$, please find the forced response by Laplace transform. (10%)

   (b) If $p = \sqrt{k/m}$, and $c = 2\sqrt{mk}$, please find the forced response by Laplace transform. (10%)

2. For the same second-order system described at ***Problem 1*** above,

   $F_0 = 10,\ m = 1,\ c = 4,\ k = 24.$

   (a) Determine *p* such that you get the steady-state response of maximum possible amplitude, named as "p_r", and this amplitude, named as " x_{max}". (10%)

   (b) Find the general solution of the second-order ODE (Ordinary Differential Equation) with "p_r". (10%)

3. Solve the BVP as shown:

   $\frac{\partial^2 u}{\partial x^2} = k\frac{\partial u}{\partial t}$, $0 < x < 10$, $t > 0$,

   Boundary conditions: u(0,t) = 10, u(10, t) = 20,

   Initial condition: u(x, 0) = 10 + x. (20%)

4. Evaluate the Cauchy principal value of the integral:

   $\int_0^\infty \frac{x\sin x}{x^2+9}\,dx$ (10%)

5. Given a circular helical curve on the surface of a cylinder $r(t) = [a\cos t\ a\sin t\ ct]$, in which *a* is the radius of the cylinder and *c* is the pitch of the circular helical curve, please find the curvature $\kappa(s) = |r''(s)|$ and torsion $\tau(s) = -p(s)\bullet b(s)$ of the circular helical curve, in which *p*(*s*) is the unit principle normal vector and *b*(*s*) is the unit binormal vector. (15%)

6. Solve the system of linear ODEs by variation of parameters (15%)

$$\begin{cases} y_1' = y_1 - 10y_2 + t \\ y_2' = -y_1 + 4y_2 + 1 \end{cases}$$