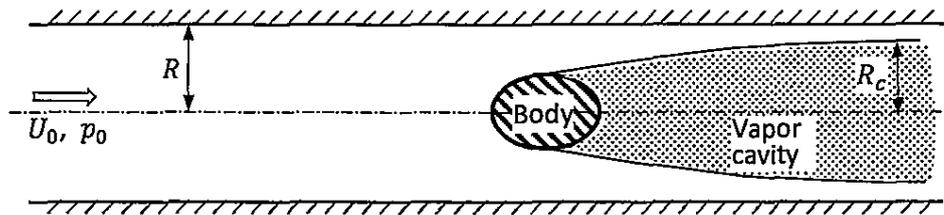


※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (20%) An axisymmetric body (a sphere if you wish) is mounted in a water tunnel that has a circular cross-section of radius, R . The velocity U_0 far upstream is fixed. It is observed that when the upstream pressure p_0 is lowered a sufficient amount, the liquid behind the body boils and forms a very long cavity of vapor at the vapor pressure p_v ($p_v < p_0$).



It is to be assumed that there is no friction and that the density of the vapor and the effect of gravity are negligible. A parameter called the cavitation number, σ , is defined as

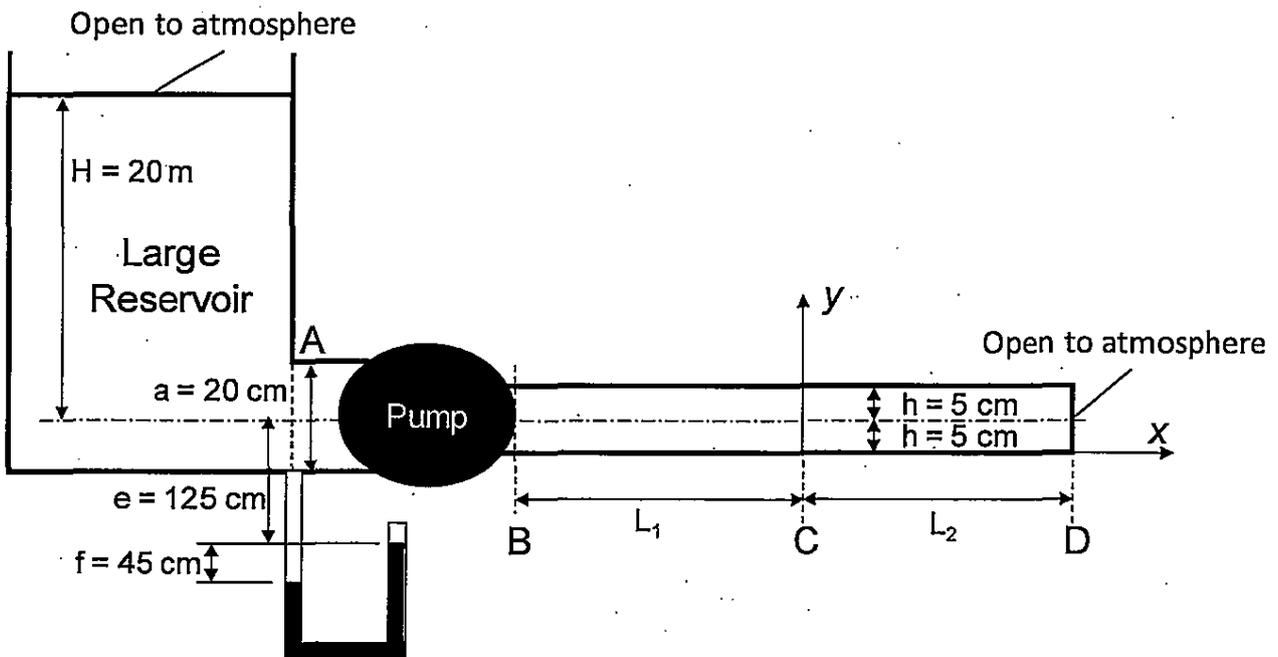
$$\sigma = \frac{(p_0 - p_v)}{\frac{1}{2}\rho U_0^2}$$

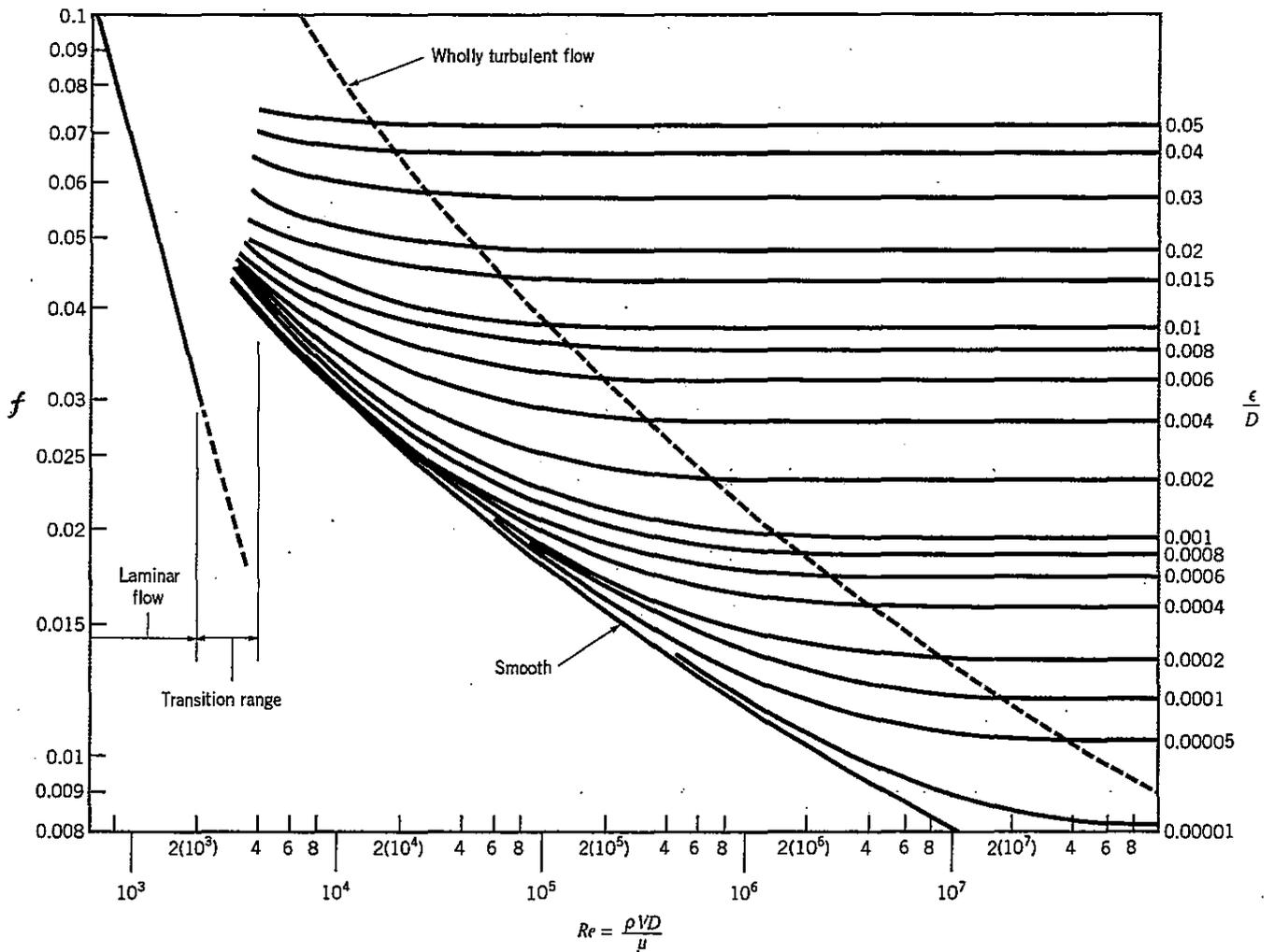
where ρ is the constant density of water.

- (a) Find the relation between R_c/R and σ for very long cavity whose asymptotic radius is R_c .
 (b) Find the drag force on the body in terms of U_0 , R , ρ , and σ .
2. (10%) A body with a typical length, L , is dragged through a viscous fluid (dynamic viscosity, μ , and density, ρ) at a velocity, U . By utilizing only the known dimensions of these quantities (in terms of kg, m and sec if you wish) construct two groupings of these quantities which have the units of force. One should contain μ but not ρ (denoted by F_V); the second should include ρ but not μ (denoted by F_I). It could then be argued that the force required to drag the body through the fluid should be related to these two "typical forces". Identify the parameter which we can use to determine the conditions under which either F_V or F_I are dominant.
3. (20%) Air enters a long horizontal ventilation duct of circular cross-section (radius 0.25m) with a velocity of 1 m/sec. At the entrance it is assumed that this velocity is uniform over the entire cross-section. However, as the flow proceeds down the duct, a thin laminar boundary layer develops on the inside wall of the duct.
 If we first assume that this is like the boundary layer on a flat plate and that the velocity away from the boundary layer remains at 1 m/sec, find the displacement thickness, δ_D (in meter), at a distance x (in meter) from the entrance. Kinematic viscosity of the air is 2.5×10^{-6} m²/sec.

Having calculated this displacement thickness we recognize that the velocity outside the boundary layer cannot remain precisely constant at 1 m/sec. Using the above-calculated δ_D find the uniform velocity outside the boundary layer at a point 200 m from the entrance. What is the pressure difference between the entrance and this point? Describe in words how you might now proceed to a more accurate boundary layer calculation which takes this pressure gradient into account.

4. (50 %) Water with density $\rho = 1000 \text{ kg/m}^3$ is flowing through the system shown in the figure. Kinematic viscosity, ν , of the water is $1.3 \times 10^{-6} \text{ m}^2/\text{s}$. The channel width, b , is 5 m from point A to D and the walls are smooth. Dimensions of the system are as shown in the figure. Velocity profiles at point A and B are assumed to be uniform. Manometer fluid of the U-tube manometer attached to the bottom wall at point A is mercury with specific gravity equals to 13.6. Boundary layers begin to grow from point B and merge at point C such that fully-developed channel flow exists between point C and D. The velocity profile in the boundary layer at point C is given as $u/U = (y/h)^{1/7}$ ($0 < y < h$) in the coordinate system shown, and the profile above the centerline is symmetric. For laminar and turbulent boundary layers, the thicknesses are $\delta/x = 5/\text{Re}^{1/2}$, and $\delta/x = 0.16/\text{Re}^{1/7}$, respectively. The drag coefficient is $C_D = 1.328/\text{Re}_L^{1/2}$ for laminar flow over a smooth flat plate with length L , and $C_D = 0.031/\text{Re}_L^{1/7}$ for turbulent flow. You may also need to use the Moody chart provided below to solve the following questions. Use 9.8 m/s^2 for gravitational acceleration.





- (a) What is the gage pressure P_A (N/m^2) at point A at the centerline?
- (b) What is the flow rate, Q (m^3/s), through the system?
- (c) If we want the same drag force from C to D (fully-developed region) and from B to C (entry region), what will be the lengths of the entry region L_1 (m) and the fully-developed region L_2 ?
- (d) What is the gage pressure at point C if $L_2 = 4.5$ m?