國立成功大學 107 學年度碩士班招生考試試題

系　所：電腦與通信工程研究所

考試科目：電磁數學　　　　　　考試日期：0205，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Consider the Bessel equation

$$x^2y'' + xy' + (x^2 - p^2)y = 0$$

   (a) (10%) Determine the solution $y(x)$ that satisfies $y(\pi) = 0$ and $y'(\pi) = 1$ when $p = 1/2$. Express your answers in terms of elementary functions.

   (b) (10%) Determine the solution $y(x)$ of above differential equation that satisfies $\lim_{x\to 0} y(x) = 0$ when $p = 2$. Is this solution unique? Explain.

   [Hint] Given limiting behaviors

$$J_p(x) \simeq \frac{1}{2^p p!}x^p \qquad \text{as } x \to 0$$

$$Y_p(x) \simeq \frac{2^p(p-1)!}{\pi}x^{-p} \qquad \text{as } x \to 0$$

2. (a) (10%) Consider the ordinary differential equation

$$(x^2 - x)y'' - (x^2 + 1)y' - (x - 1)y = 0$$

   Put this equation in the form $y'' + a_1(x)y' + a_2(x)y = 0$ and locate all its singular points. (Note: Singular points in this case are those at which $a_1(z)$ or $a_2(z)$ is not analytic.)

   (b) (10%) Find the smallest positive integer m and n such that

$$(\sqrt{3} - i)^m = (1 + i)^n$$

   (c) (10%) Determine where in the complex plane the function, $\frac{1}{e^{z-1}}$, is analytic?

3. (20%) Choose the true statement(s) from the following. (Need not to give reasons.)

   (a) Let V be a vector space, and S_1 and S_2 be two subspaces of V. It is possible that S_1 and S_2 are disjoint, $S_1 \cap S_2 = \phi$, where ϕ denotes the empty set.

   (b) Suppose that A and B are two $n \times n$ matrices. If A has an eigenvalue λ_a and B has an eigenvalue λ_b , then $(A + B)$ has an eigenvalue $(\lambda_a + \lambda_b)$.

   (c) Let T be a linear transformation (operator) on a vector space V. Then T^2 is also a linear transformation (operator) on V.

   (d) For any matrix A, we have $\text{rank}(A^TA) = \text{rank}(AA^T)$

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4. (10%) Suppose that both A and B are 3×3 matrices. The eigenvalues of A are 1, 2, and 3; while the eigenvalues of B are -2, 3, and -4. Find the determinants of AB and $A(B+I)$, respectively.

5. Suppose that A and B are two $m \times n$ matrices and $m < n$.

   (a) (10%) Is it possible that AB^T is an invertible matrix? (Give your reason.)

   (b) (10%) Is it possible that $A^T B$ is an invertible matrix? (Give your reason.)
   (We use M^T to denote the transpose of a matrix M.)