※ 考生請注意：本試題可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (30%) Consider a communication system whose transmitted signal, denoted as S, is modeled as a random variable (RV) with a zero mean and a variance of σ_s^2. The receiver is equipped with two receive (Rx) antennas. The received signal at the i-th Rx antenna is modeled as $R_i = h_i S + N_i$, i=1, 2, where h_i denotes the (real-valued) constant channel gain between the transmitter and the i-th Rx antenna, and N_i denotes the receiver noise at the i-th Rx antenna. We assume that N_1 and N_2 are independent and identically distributed, and that N_1 is of zero mean and a variance of σ_n^2. We also assume that the noise components and S are mutually independent. The receiver combines the received signals R_1 and R_2 to form the combined signal $Z = a_1 R_1 + a_2 R_2$ where a_1 and a_2 are real-valued combining weights. Note that all the RVs mentioned above are real-valued.
   (a) Determine the signal-to-noise ratio (SNR) of R_1.
   (b) Determine the SNR of Z.
   (c) If $h_1 = h_2 = 1$ and $a_1 = a_2 = 0.5$, will the SNR be improved by combining? (Compare the SNRs before and after combining.) **Why or why not?**
   (d) How would you design the combining weights? **Briefly describe your idea (e.g., the design criterion) and the motivation behind it.** [Note: (i) You do not need to solve for the combining weights. (ii) 自由發揮。Anything reasonable is OK.]

2. (10%) Will the output of a continuous-time linear time-invariant system always be periodic if the input is periodic? Justify your answer; otherwise, you will receive no credits.

3. (10%) State the differences (in at least two aspects) between the continuous-time complex exponential signal $x(t) = e^{j\omega t}$ and the discrete-time complex exponential signal $x(n) = e^{j\omega n}$.

4. (40%) Consider the receiver structure of Figure. 1, where $s_1(t)$ and $s_2(t)$ are the transmitted signals; n(t) is the zero-meaned AWGN with variance $N_0/2$; h(t) is the match filter. Please answer the following questions mathematically in detail.
   (a) (12%) Derive the optimal threshold k.
   (b) (15%) Prove that the match filter is the optimal receiver structure in terms of error rate probability.
   (c) (13%) Derive the error rate probability using the results you obtained in (b).

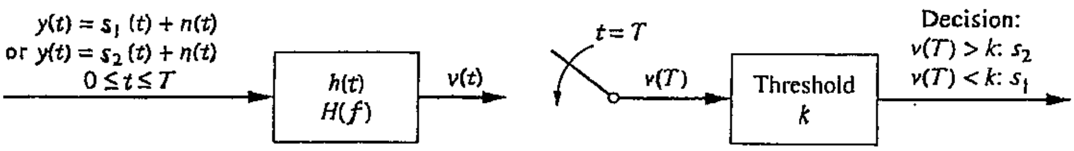

Figure 1.

5. (10%) With

$$S(f) = G(f - f_c) + G(f + f_c)$$

where

$$G(f) = \frac{A^2|P(f)|^2 + B^2|Q(f)|^2}{T_s}$$

in which $P(f)$ and $Q(f)$ are the Fourier transforms of $p(t)$ and $q(t)$, respectively. It is known that $S(f)$ is the power spectrum of the following equation.

$$x_c(t) = A[m_1(t)\cos(2\pi f_c t) + m_2(t)\sin(2\pi f_c t)] \triangleq R(t)\cos[2\pi f_c t + \theta_i(t)]$$

where $m_1(t) = d_1(t)$ and $m_2(t) = -d_2$ are random (coin toss) waveform represented as

$$d_1(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kT_s - \Delta_1)$$

and

$$d_2(t) = \sum_{k=-\infty}^{\infty} b_k q(t - kT_s - \Delta_2)$$

where $\{a_k\}$ and $\{b_k\}$ are independent, identically distributed (iid) sequences with

$$E\{a_k\} = E\{b_k\} = 0, \qquad E\{a_k a_\ell\} = A^2\delta_{k\ell}, \qquad E\{b_k b_\ell\} = B^2\delta_{k\ell}$$

in which $\delta_{k\ell} = 1$ for $k = \ell$ and 0 otherwise, is called the Kronecker delta. Please derive the power spectrum of BPSK and OQPSK.