

Problem 1.

Please use Laplace Transform to solve the following integral equation (25%)

$$y(x) = x^3 + \int_0^x \sin(x-t)y(t)dt$$

Problem 2.

Please find the general solution, including the homogeneous solution (20%) and the particular solution (5%), to the following ordinary differential equation

$$(2x+1)^2 \frac{d^2y}{dx^2} - (12x+6) \frac{dy}{dx} + 16y = 2$$

Problem 3.

Let  $\lambda_1 = 0$ ,  $\lambda_2 = 0$ ,  $\lambda_3 = 3$  be eigenvalues of a matrix  $A$  with corresponding eigenvectors  $\tilde{x}_1 = (-1, 1, 0)^T$ ,  $\tilde{x}_2 = (-1, 0, 1)^T$ ,  $\tilde{x}_3 = (1, 1, 1)^T$ . Please answer the following questions.

- (a) (15%) What is the matrix  $A$ ?  
(b) (10%) What is the rank of  $A^3 - 3A^2$ ?

Problem 4.

- (a) (15%) Please find the Fourier series of the function  $f(t) = |t|$  for  $-\pi < t < \pi$  and  $f(t + 2\pi) = f(t)$  for all  $t$ .  
(b) (10%) Please use the result to find the value of

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots = \sum_{n=1,3,5,7,\dots}^{\infty} \frac{1}{n^2}$$

試題隨卷繳回