1. Give the Cartesian equation of the hyperplane in R^4 through the origin and spanned by (1,-1,1,-1), (1,1,-1,-1), and (1,-1,-1,1). (20%)

2. Give linearly independent sets of vectors that span each of the subspaces R(A), C(A), N(A), and $N(A^T)$. $A=\begin{bmatrix}1 & 1 & 0\\ 2 & 1 & 1\\ 1 & -1 & 2\end{bmatrix}$. (20%)

3. Find a best approximation to $y = x^5$ by a straight line between x = 0 and x = 1. (20%)

4. Decide between a minimum, maximum, or saddle point for the following functions (20%)

   (a) $F=-1+4(e^x-x)-5x\sin y+6y^2$ at the point x = y = 0.

   (b) $F=(x^2-2x)\cos y$, with stationary point at x=1, y=π.

5. Decide whether the following matrices are positive definite, negative definite, semi-definite, or indefinite:

$$A=\begin{bmatrix}1 & 2 & 3\\ 2 & 5 & 4\\ 3 & 4 & 9\end{bmatrix},\quad B=\begin{bmatrix}1 & 2 & 0 & 0\\ 2 & 6 & -2 & 0\\ 0 & -2 & 5 & -2\\ 0 & 0 & -2 & 3\end{bmatrix},\quad C=-B,\quad D=A^{-1}.$$

Is there a real solution to $-x^2-5y^2-9z^2-4xy-6xz-8yz=1$? (20%)

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