

考 試 科 目	數理統計學	系 所 別	統計學系	考 試 時 間	2 月 2 日(五) 第 三 節
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1. (50%) (Sufficiency principle)

- (a) (10%) Define and interpret a sufficient statistic.
- (b) (5%) Does a sufficient statistic always exist? Prove or disprove by giving a counterexample.
- (c) (10%) Consider the following logistic regression model,

$$Y|x \sim \text{Bernoulli}(\pi(x)), \text{ where } \pi(x) = \frac{1}{1 + \exp\{-(\alpha + \beta x)\}},$$

and x is some pre-determined design point. Given a random sample of size n , $\{(x_i, y_i), i = 1, \dots, n\}$, find a sufficient statistic of (α, β) .

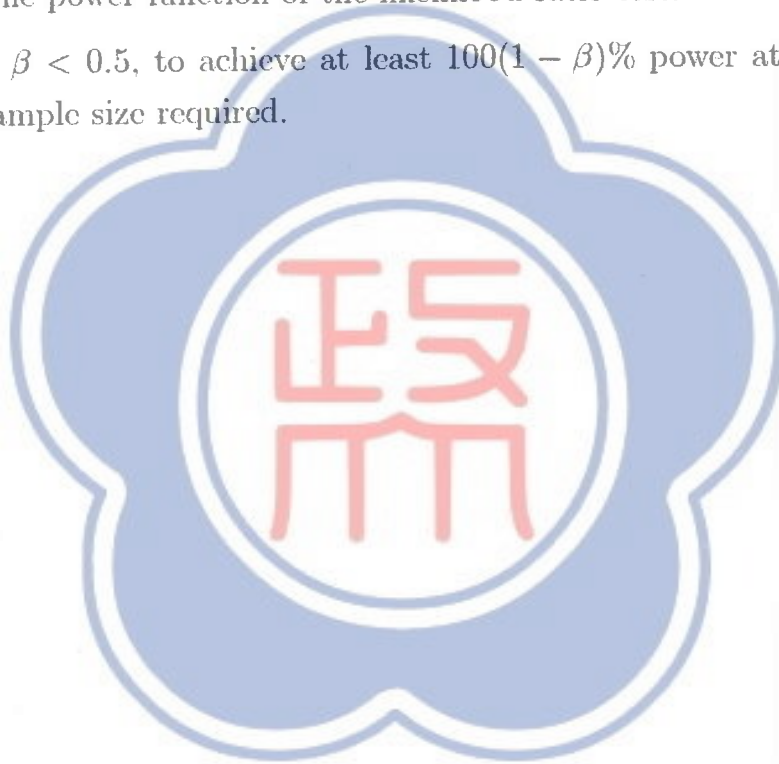
- (d) (10%) Define and interpret the **completeness** of a statistic.
- (e) (5%) Give a counterexample that a statistic is not complete.
- (f) (10%) In the era of Big Data, **discuss the importance** of the sufficiency criterion and the completeness criterion, respectively.
2. (30%) Consider a random sample of size n from a Bernoulli distribution with success probability, $0 < p < 1$.
- (a) (10%) Find a UMVUE of p , denoted by \hat{p} .
- (b) (10%) Consider squared error loss $L(t; p) = (t - p)^2$ and a uniform prior density $p \sim \text{UNIF}(0, 1)$. Find the Bayes estimator of p , denoted by \tilde{p} .
- (c) (10%) Compare \hat{p} and \tilde{p} under the squared error loss.

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3. (20%) Suppose a random sample, X_1, \dots, X_n , from $N(\mu, \sigma^2)$, where σ^2 is known. Consider testing the following hypotheses:

$$H_0: \mu = 0 \quad \text{V.S.} \quad H_1: \mu > 0.$$

- (a) (10%) Construct the likelihood ratio test at significance level α ($\alpha < 0.5$).
- (b) (5%) Find the power function of the likelihood ratio test.
- (c) (5%) Given $\beta < 0.5$, to achieve at least $100(1 - \beta)\%$ power at $\mu = 2$, find the minimum sample size required.



備

註

- 一、作答於試題上者，不予計分。
二、試題請隨卷繳交。