考試科目	近代物理	系所別	應用物理研究所	考試時間	2月3日(六)第二節

1. [*45 points*] **Short answer questions**

[5] (a) Two particles of the same mass m and moving at the same speed $v = 0.866c$ (c is the speed of light) collide head-on and combine to produce only a new particle of mass M. What is the mass of this new particle? (Note: At $v = 0.866c$, $\sqrt{1 - v^2/c^2} = 0.50$).

[5] (b) Consider two monochromatic (single-wavelength) light sources (light bulbs) emitting light of respective wavelengths λ_1 and λ_2, with $\lambda_2 > \lambda_1$. The two bulbs are otherwise identical and emit light with exactly the same intensity (in W/m^2). A detector placed a distance d from bulb 1 (emitting at wavelength λ_1) records N photons per second. When the same detector is placed at the same distance d from bulb 2 (emitting at wavelength λ_2), is the number of photons per second recorded by the detector greater than N, smaller than N, or equal to N? EXPLAIN YOUR ANSWER.

[5] (c) Some stars are observed to be reddish, and some are blue. Which stars have the higher surface temperature? Explain.

[5] (d) An electron is trapped in a one-dimensional region of space between two rigid walls at $x = 0$ and $x = L$. In the first excited state, where would you expect that the electron is most likely found?

[5] (e) A particle in an infinite square well has as its initial wave function an equal mixture of he first three orthonormal stationary states:

$$\Psi(x, 0) = A\left[\varphi_1(x) + \varphi_2(x) + \varphi_3(x)\right].$$

What is the value of the normalization constant A?

[6] (f) (i) Write out the electronic configuration for oxygen ($Z = 8$). (ii) Write out the values for the set of quantum numbers n, ℓ, m_ℓ, and m_s for each of the electrons in oxygen.

[9] (g) Consider a collection of 4 *identical* atoms obeying the rules of quantum mechanics. The atoms can occupy a set of energy levels at 2 eV, 4 eV, 6 eV and 8 eV. At a temperature of $T = 0\,\mathrm{K}$, what would be the *average energy* of these 4 atoms if they behaved like:

(i) spin-1 particles,

(ii) spin-1/2 particles,

(iii) spin-1/2 particles in a strong magnetic field in which all the electron spins point in the same direction.

Give your answers in units of eV.

[5] (h) A certain insulator has an energy gap of 6.0 eV. Is this insulator opaque or transparent to visible light. EXPLAIN YOUR ANSWER.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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2. [*15 points*] **Relativity**

An observer in frame S sees lightning simultaneously strike two points 100 m apart. The first strike occurs at $x_1 = y_1 = z_1 = 0$ at time $t_1 = 0$ and the second at $x_2 = 100\,\text{m}$, $y_2 = z_2 = 0$ at time $t_2 = 0$.

(a) What are the coordinates of these two events in a frame S' moving in the standard configuration at $0.70c$ relative to S?

(b) How far apart are the events in S'?

(c) Are the events simultaneous in S'? If not, what is the difference in time between the events, and which event occurs first?

3. [*20 points*] **The quantum oscillator**

The wavefunction

$$\psi(x) = Cxe^{-\alpha x^2}$$

describes a state of the quantum oscillator in a potential energy $U(x) = m\omega^2 x^2/2$.

(a) Using Schrödinger's equation, obtain an expression for α in terms of the oscillator mass m and the frequency ω.

(b) What is the energy of this state?

4. [*20 points*] **Expectation value**

An electron is described by the wavefunction

$$\psi(x) = \begin{cases} 0 & \text{for } x < 0 \\ Ce^{-x}(1 - e^{-x}) & \text{for } x > 0, \end{cases}$$

where C is a constant.

(a) Find the value of C that normalizes ψ.

(b) Where is the electron most likely to be found; that is, for what value of x is the probability for finding the electron largest?

(c) Calculate the average position $\langle x \rangle$ for this electron.

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