考　試　科　目	資料結構及演算法	系 所 別	資訊科學系	考 試 時 間	2 月 3 日（星期六）第一節

1. (10%) Design a polynomial-time algorithm for the following task.

   Input: A graph G.

   Output: "Yes", if G has a cycle;

   　　　　"No", if G does not have a cycle.

2. (10%) Compare arrays with linked lists.

   a. (5%) Give one advantage of linked lists over arrays.

   b. (5%) Give one advantage of arrays over linked lists.

3. (10%) Explain the reason that the worst-case running time of quick sort is $\theta(n^2)$, where n is the number of elements to be sorted.

4. (10%) Explain the reason that the worst-case running time of merge sort is $O(n \log n)$, where n is the number of elements to be sorted.

5. (10%) Often there are multiple shortest paths between two vertices of a graph. Design a polynomial-time algorithm for the following task:

   Input: An undirected graph G, where all edges have the same length, and vertices v and w in G.

   Output: The number of distinct shortest paths from v to w.

6. (10%) Design a polynomial-time algorithm to compute the maximum spanning tree, i.e., the spanning tree that has the largest total edge weight.

7. (15%) A contiguous subsequence of a list S is a subsequence made up of consecutive elements of S. For instance, if S is 5, 15, −30, 10, −5, 40, 10, then 15, −30, 10 is a contiguous subsequence but 5, 15, 40 is not. Design a polynomial-time algorithm for the following task:

   Input: A list S of numbers $a_1, a_2, \ldots, a_n$.

   Output: The contiguous subsequence of S of maximum sum.

   Note that a subsequence of length zero has sum zero.

   For the preceding example, the answer would be 10, −5, 40, 10, with a sum of 55.

8. (5%) What is a min heap? Give an example.

9. (10%) Answer the following questions about balanced binary search trees.

   a. (5%) What is a balanced binary search tree?

   b. (5%) Give one advantage of balanced binary search trees over binary search trees.

10. (10%) Let A be a problem that belongs to NP. For each of the following statements, explain whether or not the statement is true.

    a. (5%) There is no polynomial-time algorithm for A.

    b. (5%) If A can be solved in polynomial time, then P=NP.

備　　　　　　註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。