考試科目	統計學 21613	所別	經濟學系	考試時間	2 月 28 日(日) 第三節

注意事項:

(1) 請依題號順序作答。

(2) 不可使用計算機。

(3) 答題若過程錯誤 (或沒有過程) 但答案正確, 將以「零分」計算。

1. (25%) The joint probability density function (pdf) of X and Y is given by

$$f_{X,Y}(x,y) = \frac{e^{-Q/2}}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}}, -\infty < x < \infty, -\infty < y < \infty,$$

where

$$Q = \frac{1}{1-\rho^2}\left[\frac{(x-\mu_x)^2}{\sigma_x^2} - 2\rho\frac{(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y} + \frac{(y-\mu_y)^2}{\sigma_y^2}\right],$$

$$\mu_x = \mathbb{E}[X],\ \mu_y = \mathbb{E}[Y],\ \sigma_x^2 = \text{Var}(X),\ \sigma_y^2 = \text{Var}(Y),\ \rho = \frac{\text{Cov}(X,Y)}{\sigma_x\sigma_y}.$$

(1) (5%) If someone claims that above random variables X and Y **independent** when $\rho = 0$. Is she/he wrong? (You should clearly write down the reason.)

(2) If $\mu_x = 22.7$, $\sigma_x^2 = 17.64$, $\mu_y = 22.7$, $\sigma_y^2 = 12.25$, $\rho = 0.78$.

a. (5%) Find $\mathbb{E}[Y|X = x]$.

b. (5%) Find $\mathbb{P}(18.5 < Y < 22.5|X = 25)$.

(3) (10%) Assume that $\sigma_x^2 = 100$ and μ_x is unknown. If we are going to test H_o : $\mu = 60$ against H_1 : $\mu > 60$ with a observed sample mean $\bar{x} = 62.75$ based on 52 observations. What is the associated $p-$value? And what is your decision?

2. (15%) A random sample of size n is taken from a population with the probability density function (pdf):

$$f_X(x) = \frac{1}{\theta}x^{(1-\theta)/\theta},\ \ 0 < x < 1,\ 0 < \theta < \infty$$

(1) (5%) Find the **method of moments estimator** for θ, $\widehat{\theta}_{mm}$ say.

(2) (5%) Find the **maximum likelihood estimator** for θ, $\widehat{\theta}_{mle}$ say.

(3) (5%) Is $\widehat{\theta}_{mle}$ an unbiased estimator for θ? (You should clearly write down the details for your answer.)

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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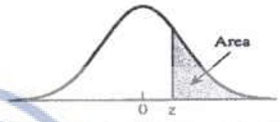
3. (10%) Let X have the cumulative distribution function (cdf) $F_X(x)$ of the continuous type that is **strictly increasing** on the support $a < x < b$. Define the new transformed random variable, Y, as $Y = F_X(X)$. What is the distribution of Y?

Normal Curve Areas
Standard normal probability in right-hand tail
(for negative values of *z*, areas are found by symmetry)



z	Second decimal place of z: .00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.00135									
3.5	.000 233									
4.0	.000 031 7									
4.5	.000 003 40									
5.0	.000 000 287									

From R. E. Walpole, *Introduction to Statistics* (New York: Macmillan, 1968).

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4. (30%) Consider the linear regression model

$$y_i = \beta_1 + \beta_2 x_i + \beta_3 z_i + e_i$$

with the nine observations on y_i, x_i and z_i given as follows

$$y = (1, 2, 3, -1, 0, -1, 2, 1, 2),$$

$$x = (0, 1, 2, -2, 1, -2, 0, -1, 1),$$

$$z = (1, -2, 1, 0, -1, -1, 1, 1, 0).$$

Please answer the following questions.

(a) Find the least square estimates of β_2 and β_3.

(b) Find the standard error for the least square estimator of β_2.

(c) Compute R^2.

(d) Find the value of the F-statistic for testing $H_0 : \beta_2 = \beta_3 = 0$.

(e) Find the value of the t-ratio statistic (t-value) for testing $H_0 : \beta_2 = 1$.

5. (20%) Consider a simple linear regression model

$$y_i = \alpha_1 + \alpha_2 x_i + e_i$$

where the e_i are independent errors with $E(e_i) = 0$ and $var(e_i) = \sigma^2 x_i^2$. Suppose that we have the following five observations

$$y = (4, 3, 1, 0, 2), \quad x = (2, 1, 1, 1, 2).$$

Please find the generalized least square estimates of α_1 and α_2.

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