

大同大學 100 學年度研究所碩士班入學考試試題

考試科目：工程數學

所別：通訊工程研究所 甲組

第 全 頁

註：本次考試 不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算器。

1. [10 points] Consider the matrix $\begin{bmatrix} 1 & 3 & -1 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$. Is it invertible? If yes, find the inverse of this matrix.

2. [24 points] Consider the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -3 & 5 & 2 \end{bmatrix}$.

- (1) Find the eigenvalues of this matrix.
- (2) Find the corresponding eigenvectors.
- (3) Is this matrix diagonalizable? Why or why not?

3. [16 points] A line $y = 2x + 1$ is mapped by the matrix $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ into another line. Find the equation of another line.

4. [15 points] An experiment can result in one or both of events A and B with the probabilities: $P(A \cap B) = 0.34$, $P(A \cap B') = 0.15$, $P(A' \cap B) = 0.46$, and $P(A' \cap B') = 0.05$. (The complement of event A is denoted by A' .)

- (1) Are events A and B independent? Explain.
- (2) Find $P(A|B)$.
- (3) Find $P(A \cup B)$.

5. [15 points] Cars arrive in a parking lot according to a Poisson process at a mean rate of 20 cars per hour.

- (1) What is the probability that the waiting time until the first arrival is more than 5 minutes?
- (2) What is the mean time until the first arrival?
- (3) What is the median time until the first arrival?

6. [20 points] The joint distribution of two random variables X and Y is known to be

$$f(x, y) = c \cdot \frac{1}{4^{x+y}}, \quad \text{for } x = 0, 1, 2, \dots \text{ and } y = 0, 1, 2, \dots$$

Find the constant c and the correlation coefficient of X and Y .