大同大學 100 學年度研究所碩士班入學考試試題

考試科目:基本數學　　　　所別:資訊工程研究所　　　　

註:本次考試 不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算器。

Part I. Linear Algrbra [50 points]

True or false (1-8, 24%), with a reason if true or a counterexample if false.

1. A and B are square matrices. If A is not invertible then AB is not invertible.
2. If the eigenvalues of A are 0,0,3, then the matrix is certainly not diagonalizable.
3. The determinant of –A is –|A|.

(Questions 4-8) Each of the following is a subspace:

4. All vectors x in R^3 such that $x^T\begin{bmatrix}1\\2\\3\end{bmatrix}=0$.
5. All vectors (x,y) in R^2 such that $x^2-y^2=0$.
6. All vectors (x,y) in R^2 such that $x+y=2$.
7. All vectors x in R^3 that are in the column space AND in the nullspace of the matrix $\begin{bmatrix}1&-2&1\\1&-2&1\\1&-2&1\end{bmatrix}$.
8. All vectors x in R^3 that are in the column space OR in the nullspace of the matrix $\begin{bmatrix}1&-2&1\\1&-2&1\\1&-2&1\end{bmatrix}$.

9. [3%] The complete solution to Ax = b is

$$x=\begin{bmatrix}1\\0\\-1\end{bmatrix}+c\begin{bmatrix}1\\1\\0\end{bmatrix}+d\begin{bmatrix}-2\\0\\1\end{bmatrix}, c,d\in R$$

If A is an $m\times n$ matrix with rank r, which of the following are valid choices of (m, n, r)?
(a) (1,3,1) (b) (2,3,2) (c) (3,3,2) (d) all of the above (e) none of the above.

10. [4%] Suppose that A is the matrix $\begin{bmatrix}2&1\\6&5\\2&4\end{bmatrix}$. Is the vector $b=\begin{bmatrix}8\\28\\14\end{bmatrix}$ in the column space of A? Explain your answer.

11. For the following two matrices,

$$A=\begin{bmatrix}1&1&1&1\\1&1&1&1\\1&1&1&0\\1&1&0&0\end{bmatrix}, B=\begin{bmatrix}0&1&1&1\\1&1&1&1\\1&1&1&0\\1&1&0&0\end{bmatrix}$$

Find $detA$ [3%].
Find the cofactor C_{11} of B and then find $detB$ [4%].

12. When $a+b=c+d$, show that (1,1) is an eigenvector [3%]. Find both eigenvalues. [4%]

$$A=\begin{bmatrix}a&b\\c&d\end{bmatrix}$$

13. Suppose a linear transformation T maps (1,1) to (2,2) and (2,0) to (0,0). Find T(0,2). [5%]

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Part II. Discrete Math [50 Points]

True (T) or False (F) [20%]:
2 points for each correct answer, and –1 point for each wrong answer. Be careful.

1. $(p \wedge (\neg q)) \rightarrow (p \rightarrow q)$ is a tautology.
2. Both $\sqrt{2}$ and π are irrational numbers.
3. If $\sqrt{2}$ is rational then π is rational.
4. If n is an integer such that n^2 is divisiable by 4 then n is divisible by 4.
5. Language $a^n b^n$ is regular.
6. Language $a^n b^n$ cannot be accepted by a finite state machine.
7. Every infinite set contains a countably infinite subset.
8. If A is countably infinite, so is 2^A.
9. Let A, B, and C be three *pairwise independent* random variables. Then, Prob($A \cap B \cap C$) = Prob(A)Prob(B)Prob(C).
10. If A, B, and C are any three finite sets, and $A \cap B \cap C = \varnothing$, then $|A \cup B \cup C| \leq |A| + |B| + |C|$.

Muliple Choice [30%]:
Each of the following questions has axactly one correct choice. 2 points for each correct choice, and –0.5 point for each wrong choice. Be careful.

1. How many integral solutions are there of $x_1 + x_2 + x_3 + x_4 + x_5 = 50$ where $x_i \geq 0$
   (a) $\binom{55}{5}$;
   (b) $\binom{55}{4}$;
   (c) $\binom{54}{5}$;
   (d) $\binom{54}{4}$;
   (e) none of the above.

2. Answer problem 1 for $x_i \geq 1$
   (a) $\binom{49}{5}$;
   (b) $\binom{49}{4}$;
   (c) $\binom{50}{5}$;
   (d) $\binom{50}{4}$;
   (e) none of the above.

3. A binary relation R is called Partial order relation if
   (a) it is reflexive and transitive;
   (b) it is symmetric and transitive;;
   (c) it is reflexive, symmetric and transitive;
   (d) it is reflexive, antisymmetric and transitive;
   (e) none of the above.

4. How many functions are there from a set with three elements to a set with two elements?
   (a) 6
   (b) 8
   (c) 10
   (d) 12
   (e) none of the above.

5. Which of the following statements is true?
   (a) If f_1 and f_2 are $O(g)$, then $f_1 f_2$ is $O(g)$.
   (b) If f is $O(g)$, then f is $O(g/2)$.
   (c) If f is $O(g)$, then g is $O(f)$.
   (d) If f is $O(g)$, then g is not $O(f)$.
   (e) none of the above.

6. If a and b are any positive integers with $b\neq 0$ and q and r are non negative integers such that $a = bq + r$ then
   (a) $\gcd(a, b)=\gcd(b, r)$
   (b) $\gcd(a, r)=\gcd(b, r)$
   (c) $\gcd(a, q)=\gcd(q, r)$
   (d) $\gcd(a, b)=\gcd(a, r)$
   (e) none of the above.

7. According to De Morgan's laws, $\overline{A\cap(B\cup C)}=?$
   (a) $\overline{A}\cap(B\cap C)$;
   (b) $\overline{A}\cup(B\cap C)$;
   (c) $\overline{A}\cup(\overline{B}\cap\overline{C})$;
   (d) $\overline{A}\cup(\overline{B}\cup\overline{C})$;
   (e) none of the above.

8. Which random variable of the following distributions is not a discrete one?
   (a) exponential disribution;
   (b) geometric distrubution;
   (c) hypergeometric distribution;
   (d) binomial distribution;
   (e) none of the above.

9. Suppose that random variable X has the following with probability mass function:

   $$p_X(x)=\frac{5!}{x!(5-x)!}(0.7)^x(0.3)^{5-x}, x=0,1,\ldots 5$$

   Then, the mean of random variable X is
   (a) 3.5;
   (b) 1.5;
   (c) .7;
   (d) .21;
   (e) none of the above.

10. The number of telephone calls that pass through a switchboard has a Poisson distribution with mean equal to 2 per minute. The expected number of phone calls that pass through the switchboard in one minute is
   (a) 4;
   (b) 3;
   (c) 2;
   (d) 1;
   (e) none of the above.

11. Refer to question 10. The probability that no telephone calls pass through the switchboard in two consecutive minutes is
   (a) 0.2707;
   (b) 0.1535;
   (c) 0.0183;
   (d) 0.0366;
   (e) none of the above.

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12. A multiple choice test has 15 questions, with each question having 5 possible answers. Suppose a student randomly guesses the answer of each question. What is the probability that the student will answer all 15 questions correctly?
    (a) 1/5.
    (b) $1/5^{15}$
    (c) $1/C_5^{15}$
    (d) $1/C_5^{19}$
    (e) none of the above.

13. $\binom{100}{0}+\binom{100}{0}\cdot 2+\binom{100}{0}\cdot 2^2+\cdots+\binom{100}{99}\cdot 2^{99}+\binom{100}{99}\cdot 2^{100}=?$
    (a) 3^{100};
    (b) $2^{101}+1$;
    (c) 3^{101};
    (d) 3^{99};
    (e) none of the above.

14. Suppose we have a bit value of 0 with probability p, and a value of 1 with probability $1-p$. A measure of the randomness of this event is $H(p)=-p\log_2 p-(1-p)\log_2(1-p)$. Find the value for p which makes $H(p)$ maximum.
    (a) $p=0$;
    (b) $p=1$;
    (c) $p=0.5$
    (d) $p=\sqrt{2}/2$;
    (e) none of the above.

15. Determine the number of strings that can be formed by rearranging the letters *SCHOOL*? (Note: Be careful, there are two *O*'s!)
    (a) 5!;
    (b) 6!/2!;
    (c) 6!·2!;
    (d) 4!·2!;
    (e) none of the above.