

1. Why do we specify that the foil be thin experimentally intended to check the Rutherford scattering formula? Explain why the scattering of  $\alpha$  particles at very small angles disagree with the Rutherford formula? (10%)
2. What is space quantization? What is exchange force? (10%)
3. A 100-keV photon collides with an electron at rest. It scattered through  $90^\circ$ . What is its energy after the collision? What is the kinetic energy in eV of the electron after the collision, and what is the direction of its recoil? The electron mass is  $0.511 \text{ MeV}/c^2$ . (15%)
4. Consider a system of three non-interacting identical spin  $\frac{1}{2}$  particles that are in the same spin up state and confined to move in a one-dimensional infinite potential well of length  $a$ :  $V(x) = 0$  for  $0 < x < a$  and  $V(x) = \infty$  for other values of  $x$ . Determine the energy and wave function of the ground state, the first excited state, and the second excited state. (15%)
5. The Hamiltonian for a certain three-level system is represented by the matrix

$$\hat{H} = \hbar\omega \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \quad (1)$$

Two other observables,  $A$  and  $B$ , are represented by the matrices

$$\hat{A} = \lambda \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad \hat{B} = \mu \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad (2)$$

where  $\omega, \lambda, \mu$  are positive real numbers.

- (a) (15%) Find the eigenvalues and (normalized) eigenvectors of  $\hat{H}$ ,  $\hat{A}$ , and  $\hat{B}$ .
- (b) (15%) Suppose the system starts out in the generic state

$$|S(0)\rangle = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}, \quad (3)$$

with  $|c_1|^2 + |c_2|^2 + |c_3|^2 = 1$ . Find the expectation value (at  $t = 0$ ) of  $\hat{H}$ ,  $\hat{A}$ , and  $\hat{B}$ .

- (c) (20%) What is  $|(S)(t)\rangle$ ? If you measured the energy of this state (at time  $t$ ), what values might you get, and what is the probability of each? Answer the same questions for  $A$  and for  $B$ .