

逢甲大學100學年度碩士班招生考試試題 編號：073 科目代碼：236

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| 科目 | 工程數學 (微分方程、拉氏轉換、線性代數) | 適用系所 | 電子工程學系固態電子組、電路與系統組 | 時間 | 100分鐘 |
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※請務必在答案卷作答區內作答。

一、 $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 3 & 1 \\ -3 & 1 & -1 \end{bmatrix}$ (20%)

Find a matrix P and D , such that $A = PDP^{-1}$, $D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$ $\lambda_1 < \lambda_2 < \lambda_3$

二、 Find the standard matrix for the linear Transformation $T : R^3 \rightarrow R^2$ defined by $T(x, y, z) = (2x + y, x - 3y)$ (10%)

三、 1. Find the inverse Laplace transform of $\frac{s \cdot e^{-s}}{s^2 + 9}$ (10%)

2. Solve the following by Laplace transform for (10%)
 $y''(t) - 4y'(t) + 3y(t) = e^{2t}$, $y(0) = 0$, $y'(0) = 0$

四、 Consider the following wave-equation: (40%)

$$\frac{\partial^2 u(x, t)}{\partial t^2} = 25 \frac{\partial^2 u(x, t)}{\partial x^2}$$

1. Solve the equation by separation of variable method (變數分離法) (10%)

With boundary conditions: $u_x(0, t) = 0$, $u_x(5, t) = 0$

Initial conditions: $u(x, 0) = 2 + 3 \cos 2\pi x$, $u_t(x, 0) = 4 + 5 \cos 5\pi x$

2. Solve the equation by separation of variable method (變數分離法) (15%)

With boundary conditions: $u_x(0, t) = 0$, $0 < x < \infty$

Initial conditions: $u(x, 0) = e^{-2x}$, $x > 0$, $u_t(x, 0) = e^{-5x}$, $x > 0$

3. Solve the equation by using the Laplace transform with respect to t (15%)

With initial conditions: $u(x, 0) = u_t(x, 0) = 0$, $0 < x < \infty$, $t > 0$

Boundary conditions:

$$u(0, t) = 1 + t - 4 \int_0^t (t - \tau) u(0, \tau) d\tau, t \geq 0, \text{ as } x \rightarrow \infty u(x, t) \text{ is bounded}$$

五、 If $y_1(x) = x$ is a solution of the following differential equation (10%)

$$x^2 y'' - xy' + y = 0$$

Please determine the second solution $y_2(x)$ by variation of parameters method (參數變易法) such that the solution of the equation is $y(x) = c_1 y_1(x) + c_2 y_2(x)$