逢甲大學100學年度碩士班招生考試試題 編號：069 科目代碼：234

科目	工程數學	適用系所	電機工程學系電波組、光電組	時間	100分鐘

※請務必在答案卷作答區內作答。

一、Solve the following initial value problem. (20%)

1. $xdx+(x^2y+4y)dy=0, \quad y(4)=0.$

2. $\frac{d^2x}{dt^2}+2\frac{dx}{dt}+\omega^2x=\cos\gamma t;\ \ x(0)=0,\ \ x'(0)=0,$ where ω and γ are constants. $\omega\neq\gamma,\ \ \omega>1.$

二、Use the method of Frobenius to obtain two linearly independent series solutions of the given differential equation about x=0. Form the general solution on (0,∞). (15%)

$$3xy''+(2-x)y'-y=0$$

三、Use the Laplace transform to solve the given initial-value problem. (15%)

$y''+y=f(t),\ \ y(0)=1,\ \ y'(0)=0$

where

$$f(x)=\begin{cases}0, & 0\le t<\pi/2\\ \sin t, & t\ge\pi/2\end{cases}$$

四、Consider the following heat-equation: (35%)

$$\frac{\partial u}{\partial t}=16\frac{\partial^2 u}{\partial x^2}$$

1. Solve the equation by separation of variable method (變數分離法) (15%)
   with boundary conditions: $u_x(0,t)=0, u_x(4,t)=0$
   initial condition: $u(x,0)=2+2\cos 2\pi x+3\cos 3\pi x$

2. Solve the equation by separation of variable method (變數分離法), for $-\infty<x<\infty$ (10%)
   Initial condition: $u(x,0)=\begin{cases}xe^{-x}, & x>0\\ 0, & x<0\end{cases}$

3. Solve the equation by Fourier **cosine** transform in x (10%)
   with boundary condition $u_x(0,t)=0,\ 0<x<\infty$
   initial condition: $u(x,0)=\begin{cases}1, & 0<x<1\\ 0, & x>1\end{cases}$

五、If $f(x)=\cos 4x$ and $g(x)=[H(x+3)-H(x-5)]$, where $H(t)$ is the unit-step function. (15%)

1. Find Fourire transform of $f(x)$, $F(\omega)=F\{f(x)\}$ (5%)
2. Find Fourire transform of $g(x)$, $G(\omega)=F\{g(x)\}$ (5%)
3. Find Fourire transform of $h(x)=f(x)g(x)$, $H(\omega)=F\{f(x)g(x)\}$ (5%)