

科目	離散數學	適用系所	資訊工程學系	時間	100分鐘
----	------	------	--------	----	-------

※請務必在答案卷作答區內作答。

1. (10 %) Please verify the stated logical equivalences.

(a)  $p \vee q \equiv \neg(\neg p \wedge \neg q)$ .

(b)  $\neg(p \oplus q) \equiv p \leftrightarrow q$ .

2. (10%) Which of the relations  $\in, \subset, \subseteq, =$  can replace the symbol  $\triangleleft$  in (a) and (b)?

(a)  $\{9\} \triangleleft \{9, \{9, 10\}\}$ .

(b)  $\{3\} \triangleleft \{3, \{3\}, \{\{3\}\}\}$ .

3. (10%) Solve the following recurrence:

$$\begin{cases} f(n) = 5f(n-1) - 6f(n-2) + 1, n \geq 2 \\ f(0) = 1 \\ f(1) = 1 \end{cases}$$

Show your derivation in detail.

4. (10%) (a) What is the number of permutations of the letters in the word TERMINALS?

- (b) What is the number of permutations of the letters in the word MESSAGES?

- (c) In (b), what is the number of permutations when all S's are together?

5. (10%) A sequence of numbers  $a_1, a_2, a_3, \dots$  is defined by (1)  $a_1 = 1, a_2 = 2$  (2)

$$a_n = a_{n-1} + a_{n-2}, n \geq 3. \text{ (a) Determine the values of } a_3, a_4, a_5, a_6, \text{ and } a_7. \text{ (b) Prove}$$

that for all  $n \geq 1$  and  $n \in \mathbb{Z}^+, a_n < (7/4)^n$ .

- 6 (10%) Prove that every amount of money of 4 dollars or more can be formed using just two-dollar bills and five-dollar bills.

- 7 (10%) Let  $R$  be the relation on the set of ordered pairs of positive integers such that  $((a, b), (c, d)) \in R$  if and only if  $a - b = c - d$ .

- (a) Show that  $R$  is an equivalence relation.

- (b) Find four pairs in  $[(1, 5)]$ .

- 8 (10%) Alice, a coffee lover, visits a coffee shop each day and orders one cup of coffee of either Espresso, Cappuccino, Latte, or Macchiato.

- (a) In how many ways can she order one cup of coffee each day so that she enjoys each of the 4 types of coffee at least once during a (7-day) week.

- (b) How many ways can she have if there are 7 types of coffee?

- 9 (10%) Prove that at a party where there are at least two people, there are two people who know the same number of other people there.

- 10 (10%) Construct deterministic finite-state automaton that recognize the set of (binary) bit string that contain an odd number of 1s and that end with at least three consecutive 0s.