

科目	線性代數	適用系所	應用數學系A組	時間	100分鐘
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※請務必在答案卷作答區內作答。

1. (20%) Let $T_A: R^4 \rightarrow R^4$ be defined by $T_A(x) = Ax$, where

$$A = \begin{pmatrix} -1 & 0 & -2 & -1 \\ -1 & 1 & -3 & 0 \\ 2 & 0 & 4 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}, x \in R^4,$$

find the rank of T_A .

2. (20%) Use Gaussian elimination to solve the system of linear equations

$$\begin{cases} 2x_1 + 2x_2 - x_3 + x_5 = 0 \\ -x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0 \\ x_1 + x_2 - 2x_3 - x_5 = 0 \\ x_3 + x_4 + x_5 = 0 \end{cases}$$

3. (20%) Let T is a linear transformation from R^2 to R^3 , and $T(2,3) = (-1,3,2)$, $T(-1,2) = (1,2,-1)$, find the formula of $T(x,y)$.

4. (20%) Let $A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$, find $P(A)$ where $P(x) = x^4 - 10x^2 + 9$.

5. (20%) Let $A = \begin{pmatrix} 0 & -2 \\ 1 & 3 \end{pmatrix}$, find A^n for any positive integer n .