

科目	微積分	適用系所	科技管理研究所	時間	100分鐘
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※請務必在答案卷作答區內作答。 共 2 頁第 1 頁

(一) 單選題 (50%) [每一題 5 分且都只有一個正確選項，請依題序將選項 (a~e) 填於答案卷上]

- () Which function is differentiable at $x=0$? (a) $\frac{1}{x^2}$ (b) $|x|$ (c) $\sqrt[3]{x}$ ($=x^{1/3}$)
(d) $\begin{cases} -x, & \text{if } x \leq 0 \\ x^2 - x, & \text{if } x > 0 \end{cases}$ (e) $\sqrt{|x|}$
- () The area of the region enclosed by the x -axis and the curve of a continuous function $f(x)$, from $x=a$ to $x=b$ is (a) $\int_a^b |f(x)| dx$ (b) $\int_a^b f(x) dx$ (c) $\int_a^b f(|x|) dx$
(d) $f(b) - f(a)$ (e) $F(b) - F(a)$, where $F'(x) = f(x)$.
- () For the function $f(x) = \frac{1}{x^2 - 4}$, which is wrong? (a) It has one horizontal asymptote.
(b) It has two vertical asymptotes. (c) $\int_3^\infty f(x) dx = \infty$. (d) It has a horizontal tangent line at $x=0$. (e) It has no absolute minimum on $(-\infty, \infty)$.
- () If $f''(x) = \frac{-1}{x^2}$, $f'(1) = 0$, and $f(e) = 1$, then $f(1) =$ (a) $e-1$ (b) $3-e$ (c) 1
(d) 0 (e) none of the above.
- () On the interval $(-1, 1)$, $f(x) = e^{-x^2/2}$ is (a) concave up (b) concave down (c) decreasing
(d) increasing (e) equal to $\frac{d}{dx} \left(\int_{-1}^{x/2} e^{-2t^2} dt \right)$.
- () The function $f(x) = \sqrt[3]{x^2} (x-3)$ has a local (relative) minimum at $x =$ (a) $\frac{6}{5}$ (b) 0 (c) 3
(d) 1 (e) none of the above.
- () Which of the following improper integrals is not true?
(a) $\int_1^\infty \frac{1}{x} dx$ diverges. (b) $\int_0^2 \frac{1}{(x-1)^2} dx$ converges. (c) $\int_{-1}^0 \frac{1}{x^2} dx$ diverges.
(d) $\int_1^\infty \frac{1}{x^2} dx$ converges. (e) $\int_0^1 \frac{1}{\sqrt{x}} dx$ converges.
- () The limit of the sequence $\left\{ \frac{\log_2 n}{n^{1/100}} \right\}$ is (a) ∞ (b) 1 (c) 0 (d) $1/e$ (e) none of the above
- () Which is not true? (a) $\int_1^e (\ln 3x) - \ln x dx = (e-1) \ln 3$ (b) $(\ln(x^x))' = 1 + \ln x$ (c) $\log_2 x = \frac{\ln x}{\ln 2}$
(d) $\lim_{x \rightarrow \infty} (\log_{1/2} x) = \infty$ (e) $\lim_{x \rightarrow 0^+} (\ln x) = -\infty$
- () If $y^3 - y + 2x^3 - x = 7$, then $\frac{dy}{dx}$ at the point $(x, y) = (1, 2)$ is
(a) $-5/3$ (b) 17 (c) 10 (d) $-5/4$ (e) none of the above

(二) 填充題 (30%) [每一題 5 分，請依序填下答案於答案卷上，勿列出計算過程]

1. If $y = x^\pi + \pi^x$, then $\frac{dy}{dx} = \underline{\hspace{2cm}}$.
2. $\frac{\partial^3}{\partial x \partial x \partial y}(x^3 y^5 + 3xy^4) = \underline{\hspace{2cm}}$.
3. The equation of the tangent line to the graph of $y = (\sqrt{x})^x$ at $x=1$ is $\underline{\hspace{2cm}}$.
4. $\int (3-x)(x^2 - 6x + 1)^9 dx = \underline{\hspace{2cm}}$.
5. The average value of $f(x) = x^3 \cos x$ on $[-\pi, \pi]$ is $\underline{\hspace{2cm}}$.
6. $\int_0^1 \int_x^1 e^{x/y} dy dx = \underline{\hspace{2cm}}$. (Hint: by reversing the order of integration.)

(三) 計算題 (20%) [每一題 10 分，請寫下詳細計算過程，否則不予計分]

1. Determine which is larger: π/e or $\ln \pi$? (Hint: Consider the function $f(x) = \frac{x}{e} - \ln x$)
2. Sketch the region bounded by the graphs of $x + y = 2$ and $x = y^2$ (約略的畫出此區域). Then, find the area of the region using two ways – (1) integration with respect to x and (2) integration with respect to y .