逢甲大學100學年度碩士班招生考試試題 編號：019 科目代碼：312

科目	統計學	適用系所	經濟學系、國際貿易學系、財稅學系丙組、科技管理研究所、合作經濟學系	時間	100 分鐘

※請務必在答案卷作答區內作答。

I. Multiple Choice: Select the best answer

1. (6%) Consider you observe the income distribution of family from a random sample in Taiwan. You calculate mean, mode and median. What will you expect?
a. median > mode > median
b. mean > median > mode
c. mode > median > median
d. mean > mode > median

2. (6%) We use the t-distribution to calculate a confidence interval for the population mean μ. If we double the sample size from 10 to 20, the interval would become smaller because of
a. the change in degrees of freedom.
b. the change in standard error.
c. both a and b.
d. none of the above.

3. (6%) Suppose you were to toss a fair coin 100,000 times. What is the probability that you'll get heads between 49.5% and 50.5% of the time?
a. 100%
b. 95%
c. 50%
d. none of the above

4. (6%) Which statement(s) below is (are) correct?
a. If two variables are independent, their correlation is zero.
b. A correlation is always between 0 and 1.
c. The p-value is the probability that the null hypothesis is correct.
d. Both a. and b. are correct.

5. (6%) Two events, A and B, are such that *P(A)*=0.5, *P(B)*=0.3, and $P(A \cap B) = 0.1$. Which statement(s) below is (are) correct?
a. $P(A \mid B) = \frac{1}{2}$
b. $P(B \mid A) = \frac{1}{4}$
c. $P(A \cup B) = \frac{2}{3}$
d. none of above is correct.

II. Problems

1. (20%) Consider the following simple regression without constant:

$y_t = \beta x_t + u_t$,t=1, 2

where $\beta = 2$ and x_t takes on fixed values $x_1 = 1$, $x_2 = 2$. u_t has the following discrete joint probability distribution for each value:

(u_t)	Probability
1	1/2
-1	1/2

(a) What is the probability of y=3?

(b) Find $Var(u_t)$ and $\text{cov}(u_1, u_2)$.

(c) Find the sampling distributions of the following two estimators of β

$$b_1 = \sum_{t=1}^{2} y_t \Big/ \sum_{t=1}^{2} x_t \quad \text{and} \quad b_2 = \sum_{t=1}^{2} y_t x_t \Big/ \sum_{t=1}^{2} x_t^2$$

(d) Show that var(b_1)>var(b_2).

2. (10%) Let and $X_1, X_2, ..., X_n$ be a random sample from the geometric distribution with p.m.f. $f(x;\theta) = (1-\theta)^{x-1}\theta$, where $x = 1,2,3,...$, and $0 < \theta < 1$. Derive the maximum likelihood estimator of θ.

3. (10%) If $E(X) = 17$ and $E(X^2) = 298$, please determine

(a) the lower bound for $P(10 < X < 24)$.

(b) the upper bound for $P(|X - 17| > 16)$.

4. (10%) In developing countries in Africa and the Americas, let p_1 and p_2 be the respective proportions of women with nutritional anemia. Find an approximate 90% confidence interval for $p_1 - p_2$ given that a random sample of $n_1 = 2100$ African women yielded $y_1 = 840$ with nutritional anemia and a random sample of $n_2 = 1900$ women from the Americas yielded $y_2 = 323$ women with nutritional anemia.

5. (10%) Let $Y_1 < Y_2 < Y_3 < Y_4 < Y_5$ be the order statistics of five independent observations from an exponential distribution that has a mean of $\theta = 3$. (You may leave the answers be functions of "exp".)

(a) Compute the probability that Y_4 is less than Y_5.

(b) Determine $P(1 < Y_1)$.

6. (10%) Customers arrive at a travel agency at a mean rate of 11 per hour. Assuming that the number of arrivals per hour has a Poisson distribution, give the probability that more than 10 customers arrive in a given hour.