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| 科目 | 統計學(含數理統計學) | 適用系所 | 統計學系統計與精算碩士班應用統計暨計量財務組、精算組 | 時間 | 100分鐘 |
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※請務必在答案卷作答區內作答。 共2頁 第1頁

1. (15%) Suppose that we have a sample space $S = \{E_1, E_2, E_3, E_4, E_5, E_6, E_7\}$, where E_1, E_2, \dots, E_7 denote the sample points. The following probability assignments apply: $P(E_1) = 0.05, P(E_2) = 0.20, P(E_3) = 0.20, P(E_4) = 0.25, P(E_5) = 0.15, P(E_6) = 0.10,$ and $P(E_7) = 0.05$. Let $z_{0.025} = 1.96$, and

| |
|------------------------------|
| $A = \{E_1, E_4, E_6\}$ |
| $B = \{E_2, E_4, E_7\}$ |
| $C = \{E_2, E_3, E_5, E_7\}$ |

- (3%) Find $P(A), P(B),$ and $P(C)$.
 - (3%) Find $A \cup B$ and $P(A \cup B)$.
 - (3%) Find $A \cap B$ and $P(A \cap B)$.
 - (3%) Are events A and C mutually exclusive?
 - (3%) Find B^c and $P(B^c)$.
2. (12%) Annual starting salaries for college graduates with degrees in business administration are generally expected to be between \$30,000 and \$45,000. Assume that a 95% confidence interval estimate of the population mean annual starting salary is desired. What is the planning value for the population standard deviation? How large a sample should be taken if the desired margin of error is
- (4%) \$500?
 - (4%) \$200?
 - (4%) \$100?
3. (12%) Consider the following data for two independent random samples taken from two normal populations. ($t_{9,0.05} = 1.833$)

| | | | | | | |
|-----------------|----|---|----|---|---|---|
| Sample 1 | 10 | 7 | 13 | 7 | 9 | 8 |
| Sample 2 | 8 | 7 | 8 | 4 | 6 | 9 |

- (3%) Compute the two sample means.
 - (3%) Compute the two sample standard deviations.
 - (3%) What is the point estimate of the difference between the two population means?
 - (3%) What is the 90% confidence interval estimate of the difference between the two population means?
4. (11%) A study of the educational levels of residents and their satisfaction with public security yielded the following results.

| Educational Level | Satisfaction With Public Security | | |
|------------------------------|-----------------------------------|------|---------------|
| | Satisfied | Okay | Not Satisfied |
| Did not complete high school | 6 | 86 | 48 |
| High school degree | 13 | 126 | 56 |
| College degree | 24 | 255 | 110 |

Use $\alpha = .01$ and test to determine whether satisfaction with public security is independent of the educational level of the residents.

($\chi^2_{4,0.01} = 7.78$)

5. (10%) Let X be an RV with PDF $f(x) = e^{-x}$ if $x \geq 0$, and $= 0$ otherwise. Let $Y = \sin X$. Find the PDF of Y .
6. (30%) Let (X, Y) be jointly distributed with PDF $f(x, y) = 2$, $0 < x < y < 1$, and $= 0$ otherwise.
- (a) (5%) Find $P(T > 1/3 \mid X = 1/4)$.
 - (b) (15%) Find $E(Y \mid x)$, $\text{Var}(X \mid y)$, $\text{Cov}(X, Y)$.
 - (c) (10%) Find the PDF of $X + Y$.
7. (10%) Let (X_1, X_2, \dots, X_n) be an RV such that the correlation coefficient between each pair X_i, X_j , $i \neq j$, is ρ . Show that $-(n-1)^{-1} \leq \rho \leq 1$.