

科目	微積分	適用系所	統計學系統計與精算碩士班應用統計暨計量財務組、精算組	時間	100分鐘
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※請務必在答案卷作答區內作答。 共 2 頁第 1 頁

一、單選題(50%)(每題 5 分，答錯不倒扣。請依序作答於答案卷上)

- The coordinates of a moving particle in R^2 are $(x,y)=(4\sin(t/2),2t\cos(t))$ for time $t>0$. Calculate the length of the velocity vector at time $t=\pi/2$.
(A) $\sqrt{2}$ (B) π (C) $\sqrt{\pi^2+2}$ (D) $\sqrt{\pi^2+4}$ (E) $\sqrt{2}+\pi$
- Let f be a function on $[0,\infty)$ such that $f(0)=0$, $f(40)=150$, and

$$f'(x)=\begin{cases} 2 & \text{for } 0\leq x<10 \\ 3 & \text{for } 10\leq x<20 \\ k & \text{for } x\geq 20 \end{cases}$$
 where k is a constant. Find the value of k .
(A) 2 (B) 5 (C) 10 (D) 15 (E) None of the above
- Find the value of the integral $\int_0^1 x^3(1-x)^4 dx$.
(A) 1/35 (B) 1/60 (C) 1/126 (D) 1/280 (E) None of the above
- Find the value of A such that $\int_0^B |x-A| dx$ is minimized.
(A) 0 (B) $B/2$ (C) $2B/3$ (D) B (E) None of the above
- Find the value of $\lim_{x\rightarrow 0} \frac{\tan x}{\sin 2x}$.
(A) 0 (B) 1/2 (C) 1 (D) 2 (E) The limit does not exist
- Find the value of the integral $\iint_A e^{-(x^2+y^2)/2} dx dy$ where $A=\{-\infty < x < y < \infty\}$.
(A) 1 (B) 2 (C) π (D) 2π (E) None of the above
- Find the area of the region between $y^2=4x$ and $4x-3y=4$.
(A) 3.7 (B) 4.2 (C) 4.7 (D) 5.2 (E) None of the above
- Which of the following series are convergent?
(I) $\sum_{k=1}^{\infty} (\frac{1}{k} - \frac{1}{k+1})$ (II) $\sum_{k=1}^{\infty} \ln(k)$ (III) $\sum_{k=2}^{\infty} \ln(1 - \frac{1}{k^2})$ (IV) $\sum_{k=1}^{\infty} (\frac{k!}{100^k})$
(A) I, II (B) II, III (C) I, III (D) II, IV (E) None of the above
- Find the value of the integral $\int_2^{\infty} x^2 e^{-x} dx$.
(A) e^{-2} (B) $3e^{-2}$ (C) $5e^{-2}$ (D) $10e^{-2}$ (E) None of the above
- Find the value of the derivative $\frac{d}{dx} x^x$.
(A) x^x (B) $x^x \ln(x)$ (C) $x^x(\ln(x)+1)$ (D) $x \ln(x)$ (E) None of the above

二、填充題(30%)(每空格 5 分，無須列出計算過程。請依序作答於答案卷上)

1. Let $f(x) = x^x 3^x$. Then $f'(x) =$ _____.
2. $\lim_{x \rightarrow \infty} \left(x \ln \left(\frac{x-1}{x+1} \right) \right) =$ _____.
3. $\int_0^1 \frac{dx}{\sqrt{1-x^{1/4}}} =$ _____.
4. Given that $xy^2 = \sin(x+2y)$, then $\frac{dy}{dx} =$ _____.
5. $\int_0^{\pi/2} \sin^6 \theta \cos^4 \theta d\theta =$ _____.
6. $\int_{-\infty}^{\infty} e^{-2x^2+4x} dx =$ _____.

三、計算題(20%)(每題 10 分，請依序將計算過程作答於答案卷上)

1. Evaluate $\int_{4/\sqrt{3}}^4 \frac{dx}{x^2 \sqrt{x^2-4}}$.
2. Find the area of the region enclosed by the graphs of $x+y=2$ and $y=2x^4$.
(Please sketch the curves of two functions and indicate the closed region).