科目	工程數學	適用系所	航太與系統工程學系固力組、熱流組、控制組	時間	100 分鐘

※請務必在答案卷作答區內作答。

1. (15%) Is the following function $u = x^3 - 3xy^2$ harmonic? If the answer is yes, find a corresponding analytic function $f(z) = u(x,y) + iv(x,y)$.

2. (20%) (a) Find the Fourier series expansion of $f(x) = 1 + x$ on the interval $-1 \le x \le 1$.

    (b) Show that $\dfrac{\pi}{4} = \sum_{n=1}^{\infty} \dfrac{(-1)^n}{2n+1}$.

3. (15%) Using the Laplace transform, solve the integral equation.
    $$y(t) + \int_0^t (t-\tau) y(\tau)\, d\tau = 1$$

4. (15%) Evaluate
    $$I = \iint\limits_{S} \left(7x\vec{i} - z\vec{k} \right) \cdot \vec{n}\, dA \quad \text{over the sphere} \quad S: \ x^2 + y^2 + z^2 = 4^2$$

5. (15%) Apply Picard's iteration to solve the function $y_3(x)$ in the following problem.
    $$y' = y, \quad y(0) = 1.$$

6. (20%) Find a general solution of the following equation.
    $$(x^2 D^2 - 2xD + 2)y = x^3 \sin x.$$