逢甲大學100學年度碩士班招生考試試題 編號：008 科目代碼：203

科目	統計學	適用系所	工業工程與系統管理學系A組、B組	時間	100 分鐘

※請務必在答案卷作答區內作答。　　　共2頁第1頁

1. (10%) Bowl I contains 2 white chips, bowl II contains 2 red chips, bowl III contains 2 white chips and 2 red chips, and bowl IV contains 3 white chips and 1 red chip. The probability of selection bowl I, II, III, or IV are 1/2, 1/4, 1/8, and1/8, respectively. A bowl is selected using these probabilities, and a chip is then drawn at random. Find
(a) P(W), the probability of drawing a white chip.
(b) P(I | W), the conditional probability that bowl I had been selected, given that a white chip was selected.

2. (10%) Let X have the p.d.f.

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1, \\ c/x^3, & 1 \leq x < \infty \\ 0, & \text{elsewhere.} \end{cases}$$

Find the value of c.

3. (10%) Let X have the p.d.f. $f(x) = x^2/3$, $-1 < x < 2$. Determine the p.d.f. of $Y = X^2$.

4. (10%) Let X_1, X_2 and X_3 be independent random variables with p.d.f.s : $f_1(x_1) = 3x_1^2$, $0<x_1<1$; $f_2(x_2) = 4x_2^3$, $0<x_2<1$ and $f_3(x_3) = 6x_3^5$, $0<x_3<1$. Find the p.d.f. of $Y = \text{maximum}(X_1, X_2, X_3)$.

5. (10%) Let X_1, X_2, ..., X_{20} denote a random sample of size 20 from the uniform distribution U(0, 1). Find, approximately, $P(X_1+X_2+ ...+X_{20} < 9.1)$.

$Z \sim N(0, 1)$，$P(Z > z_\alpha) = \alpha$。$z_{0.2423} = 0.697$、$z_{0.10} = 1.282$、$z_{0.05} = 1.645$、$z_{0.025} = 1.96$。

6. A TFT-LCD manufacturer suspects that its two plants produce different proportions of "grade A" panels. Samples of sizes $n_1 = n_2 = 300$ were randomly selected from a week's production of the two plants, and $y_1 = 180$ and $y_2 = 215$ panels were classified as grade A.

（1） (10%)What is the 95% confidence interval for $p_1 - p_2$?

（2） (10%)Can you conclude that the two plants produce different proportions of "grade A" panels from (1)？Explain how.

7. Suppose that the weight of the apples from an orchard are distributed normally with mean μ= 150g and standard deviation σ=20 g.

   (1) (5%)Let $\bar{x}$ be the mean of a random sample of size n = 20 from these apples. Find the expected value and the variance of $\bar{x}$.

   (2) (5%)If we randomly select 20 apples from the orchard and pack them into a box. Suppose the weight of the empty box ~N(100, 10^2). Find the expected value of the total weight of the packed box and its variance.

   (3) (5%)Suppose those apples are sold to a Japanese company for 20 yens per gram. Find the expected value of the price for each apple and its variance.

8. (15%)Suppose that in one year the number of industrial accidents X follows a Poisson distribution with mean 3.0. If each accident leads to an insurance claim of $5000, how much money would an insurance company need to keep in reserve to be 95% certain that the calamines are covered?