

東吳大學 106 學年度碩士班研究生招生考試試題

第 1 頁，共 1 頁

系級	數學系碩士班 B 組(決策科學與海量資料分析)	考試時間	100 分鐘
科目	基礎數學	本科總分	100 分

1. 20% Suppose $f(x)$ is differentiable and $c \neq 0$ is a constant, let $u(x, t) = f(x - ct)$.

(i) Show that $u(x, t)$ satisfies $\frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} = 0$.

(ii) Show that ∇u (i.e. gradient of u) parallels $(1, -c)$ everywhere.

2. 20% The volume of a cylinder with radius $r > 0$ and height $h > 0$ is $V = \pi r^2 h$.

(i) Show that $\frac{dV}{V} = \frac{2dr}{r} + \frac{dh}{h}$ (notation: dV, dr, dh are differentials of V, r, h respectively.)

(ii) The volume of a certain cylinder V is determined by measuring r and h , which will lead to a greater error in V : a 1 % error in r or a 1 % error in h ?

3. 30% (i) Find the eigenvalues of $\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$. Choose a 2×2 square matrix P so that PAP^{-1} is diagonal.

(ii) Let $f(x, y) = 3x^2 + 2xy + 3y^2$, use (i) or (ii) to show that $(0,0)$ is a minimum of f .

(iii) What is the shape of level curves of $f(x,y)$? (you have to explain your answer)

4. 10% $\int_{-\infty}^{\infty} e^{-x^2} dx = ?$

5. 20% Given a set of data in xy -plane: (x_i, y_i) , $i = 1, 2, \dots, n$, $n \geq 2$, and $x_i \neq x_j$ if $i \neq j$. If a line

$L(x) = a + bx$ minimizing $\sum_{i=1}^n [y_i - L(x_i)]^2$ (i.e., linear least squares fitting),

(i) show that
$$\begin{bmatrix} n & \sum_{i=1}^n x_i \\ \sum_{i=1}^n x_i & \sum_{i=1}^n (x_i)^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n x_i y_i \end{bmatrix}$$

(ii) Are a, b uniquely determined?