國立臺灣大學101學年度碩士班招生考試試題

1. (10%) Suppose that $f(x) = 1$ if x is rational and $f(x) = 0$ if x is irrational. Show that $f(x)$ is discontinuous at every point.

2. (10%) Suppose that $f(x) = x\sin(1/x)$ if $x \neq 0$ and $f(x) = 0$ if $x = 0$. Show that $f(x)$ is continuous at $x = 0$ but not differentiable at $x = 0$.

3. (10%) Let $\sum_{n=1}^{\infty} a_n$ be a convergent series of positive terms, where a_n is a monotone decreasing function of n. Show that $\sum_{n=1}^{\infty} 2^n a_{2^n}$ converges.

4. (7%) (7%) Find the following limits:

$$(4a)\ \lim_{(x_1,x_2)\to(0,0)} \frac{x_1^3 - x_2^3}{x_1^2 + x_2^2}. \quad (4b)\ \lim_{(x_1,x_2)\to(0,0)} \frac{x_1 x_2}{x_1^2 + x_2^2}.$$

5. (8%) (8%) Evaluate the following integrals:

$$(5a)\ \int \sin(\ln x)\,dx. \quad (5b)\ \int_0^{\ln 4} \frac{e^x}{\sqrt{e^{2x}+9}}\,dx.$$

6. (20%) Solve the differential equation:

$$(x+1)\frac{dy}{dx} - 2(x^2+x)y = \frac{e^{x^2}}{x+1}, x > -1, y(0) = 5.$$

7. (10%) Find the area of the region in the first quadrant bounded by the line $y = x$, the line $x = 2$, the curve $y = x^{-2}$, and the X-axis.

8. (10%) Find a curve through the point $(0, 0)$ whose length integral is

$$\int_1^4 \sqrt{1 + \frac{1}{4x}}\,dx.$$

試題隨卷繳回