

中原大學 100 學年度 碩士班 入學考試

3 月 19 日 15:30~17:00

電機工程學系電力能源組、電機
工程學系智慧控制組、電機工程
學系電子電路組

誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

科目：工程數學(主考範圍：線性代數、微分方程)

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可使用計算機，惟僅限不具可程式及多重記憶者

不可使用計算機

1. Let $A = \begin{bmatrix} 1 & -z & 0 \\ -z & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, where $z \in R$. Find

(a) $\det(A)$ (5%).

(b) $\det(A^{100})$ for $z=1$ (5%).

(c) the eigenvalues of A^{100} for $z=1$ (5%). (d) Jordan matrix of A^{100} for $z=1$ (5%)

(e) the solution of the system $\frac{dx(t)}{dt} = Ax(t)$, where $x(0) = [0 \ 1 \ 1]^T$ and $z=1$. (10%)

2. Let y_1 and y_2 be the functions of t . Please solve the differential equation system: (10%)

$$\begin{cases} y_1' = -3y_1 + y_2 \\ y_2' = y_1 - 3y_2 + 2e^{-4t} + u(t) \end{cases}, \text{ with } y_1(0) = y_2(0) = 0 \text{ and a unit-step input } u(t).$$

3. Let y be a function of x . Solve the following differential equation. (15%)

$$x^2 y'' + 6xy' + 6y = x, \quad x > 0.$$

4. Solve $y(t)$ for the following differential equation for a unit-step input $u(t)$. (15%)

$$\begin{cases} y' + 6 \int_0^t z dt = -2u(t) \\ y' + z' + z = 0 \end{cases}, \text{ with } y(0) = -5 \text{ and } z(0) = 6$$

5. Given a matrix equation $X = \left(\frac{X^3 + 6X}{5} \right)^{\frac{1}{2}}$, where X is a 3×3 matrix.

(a) Find two matrices (which are not diagonal) can satisfy the matrix equation. (10%)

(b) For your answer of (a), find X^6 . (10%)

6. Consider the differential equation

$$\dot{x}(t) = -x^{\frac{q}{p}}(t),$$

with $x(0) \neq 0$, p and q are odd integers satisfying $p > q > 0$. Prove that the variable

$x(t)$ converges to zero in finite time. (10%)