中原大學 100 學年度 **碩士班** 入學考試

3 月 19 日 10:30~12:00　　資訊工程學系　　誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

科目：計算機數學　　（共 3 頁第 1 頁）

□可使用計算機，惟僅限不具可程式及多重記憶者　　■不可使用計算機

注意：
(1) 務必請按順序答題、以避免誤改！
(2) 答題時請寫題號(如果不寫題號，該題就算作是沒答)。
(3) 答案是寫在答案卷上，而不是寫在此試題紙上。

1. (9 pts.) Let S be a non-empty set. Let R be a relation on S (i.e., R ⊆ S×S). Please follow the example below in answering 1a, 1b and 1c. (Note : in writing your answers to 1a, 1b and 1c, you must treat R as a set.)

Example : **R is *symmetric* if and only if** ∀x∈S, ∀y∈S, if (x, y)∈R, then (y, x)∈R.

1a. (3 pts.) R is reflexive if and only if ________________________________.
1b. (3 pts.) R is transitive if and only if ________________________________.
1c. (3 pts.) R is anti-symmetric if and only if ____________________________.

2. (27 pts.) Let s and t be two strings of English letters. s **contain-a-substring-of** t if and only if there exists a non-empty string u such that u is both a substring of s and also a substring of t. (e.g., if s is ‘abcde’, t is ‘xcyz’ and w is ‘hijk’, then s contain-a-substring-of t but s does not contain-a-substring-of w.)

2a. (3 pts.) Give an example to show that the *contain-a-substring-of* relation is not transitive. (In your answer, you must also explain why this example of yours serves to show that the *contain-a-substring-of* relation is not transitive.)
2b. (3 pts.) Give an example to show that the *contain-a-substring-of* relation is not a partial order. (In your answer, you must also explain why this example of yours serves to show that the *contain-a-substring-of* relation is not a partial order.)
2c. (3 pts.) Is it correct to write ‘**{** a, b **}**’ as ‘**(** a, b **)**’? If not, explain why. (No explanation is needed if your answer is “Yes”.)
2d. (3 pts.) Let ∅ denote the empty set. Is it correct to write ‘**{** ∅ **}**’ as ‘∅’. If not, explain why. (No explanation is needed if your answer is “Yes”.)
2e. (3 pts.) Explain why 9 is not a prime number, and also explain why 13 is a prime number.
2f. (3 pts.) Give an example to show that a directed acyclic graph may not be a tree.
2g. (3 pts.) Define a relation R on S (i.e., R ⊆ S×S) such that R is not a function. You must specify what S is and what R is. You must also explain why this example of yours serves to show that R is not a function.
2h. (3 pts.) Show that the union of two countable sets is still countable.
2i. (3 pts.) Let M = (I, S, F) be a finite state machine, where I, the input alphabet, is { w, u, v }, S, the state set, is { a, b, c, d, e, f, g }, and F, the transition function, is as specified below. For every state s in S, **Reachable**(s) is defined to be the set { t | there is a sequence of input $i_1, i_2, ..., i_n$ such that $F(s, i_1) = s_1$, $F(s_1, i_2) = s_2$, ..., $F(s_{n-1}, i_n) = t$ }. What is **Reachable**(d)?

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F :

state\input	w	u	v
a	f	e	g
b	f	c	d
c	b	f	e
d	b	e	c
e	c	e	b
f	d	c	e
g	c	a	b

3. (5 pts.) Let α and β be two arbitrary logical formulas. Use the truth table technique to show that the following formula is a tautology. (Note : '$\supset$' is "if ... then ..."; some authors write '$\supset$' as '$\Rightarrow$').

$$((\alpha \supset \beta) \wedge (\neg\alpha \supset \beta)) \supset \beta$$

4. (9 pts.) Let α and β be two arbitrary logical formulas. Use the truth table technique to show

4a. (3 pts.) whether $\neg\alpha \vee \beta$ and $\alpha \supset \beta$ are equivalent,

4b. (3 pts.) whether $\neg\alpha \vee \beta$ and $\alpha \vee \neg\beta$ are equivalent, and

4c. (3 pts.) whether $\alpha \supset \beta$ and $\alpha \vee \neg\beta$ are equivalent.

(Note : '$\supset$' is "if ... then ..."; some authors write '$\supset$' as '$\Rightarrow$').

(**You can use just one truth table if you want.**)

5. (14 pts.) Given a matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & 1 \\ 0 & 2 & 3 \end{bmatrix},$$

find the following items in sequence: **(2pts. for each item)**

5a. the dimension of **A**,　　**5b.** the determinant of **A**,

5c. the basis of range(**A**),

5d. the coordinate of [4 2 5] with respect to the above basis,

5e. the rank of **A**,　　**5f.** the null space of **A**,

5g. the nullity of **A**.

6. (14 pts.) Given a matrix

$$\mathbf{B} = \begin{bmatrix} 3 & 1 & -2 \\ -1 & 0 & 5 \\ -1 & -1 & 4 \end{bmatrix},$$

find the following items in sequence: **(2pts. for each item)**

6a. its characteristic polynomial,　　**6b.** all of its eigenvalues,

6c. the eigenvector(s) with respect to the first eigenvalue,

6d. the eigenvector(s) with respect to the second eigenvalue,

6e. its minimal polynomial,

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6f. its generalized eigenspace,　　**6g.** the Jordan canonical form of **B**.

7. (22 pts.) Given a quadratic polynomial $p(\mathbf{x} = [x_1 \quad x_2 \quad x_3]) = 3x_1^2 + 3x_2^2 + 3x_3^2 - 2x_1x_3$,
find the following items in sequence: **(2pts. for each item)**
7a. the symmetric matrix **C** associated with p(**x**)=**x'Cx**,
7b. the characteristic polynomial of **C**,　　**7c.** all of the eigenvalues of **C**,
7d. the unit eigenvector with respect to the first eigenvalue,
7e. the unit eigenvector with respect to the second eigenvalue,
7f. the unit eigenvector with respect to the third eigenvalue,
7g. its generalized eigenspace **Q**,　　**7h.** the orthonormality of **Q**,
7i. the orthonormal transformation of **C**,
7j. the transformed vector $\mathbf{x}' = [x_1' \quad x_2' \quad x_3']$ of **x** by the above transformation,
7k. the transformed quadratic form $p'(x_1', \quad x_2', \quad x_3')$.