中原大學 100 學年度 **碩士班** 入學考試

3 月 19 日 10:30~12:00　　電子工程學系系統組　　誠實是我們珍視的美德，我們喜愛「拒絕作弊，堅守正直」的你！

科目：工程數學(範圍：線性代數、機率)　　（共 2 頁第 1 頁）

□可使用計算機，惟僅限不具可程式及多重記憶者　　■不可使用計算機

1. (10%) Row reduce the following matrix to reduced echelon form, and list the pivot columns.

$$\begin{bmatrix} 1 & 4 & 7 & 10 \\ 2 & 5 & 8 & 11 \\ 3 & 6 & 9 & 12 \end{bmatrix}$$

2. (10%) Find the general solution of the system whose augmented matrix is given below.

$$\begin{bmatrix} 2 & 4 & 3 \\ -6 & 12 & 9 \\ 4 & -8 & 6 \end{bmatrix}$$

3. (10%) For $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ -2 & 4 \end{bmatrix}$, define $T : R^2 \to R^2$ by $T(\mathbf{x}) = \mathbf{Ax}$. Find the images under T of $\mathbf{u} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, and $\mathbf{u} + \mathbf{v} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$.

4. (10%) Let $\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$，$\mathbf{b}_1 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, $\mathbf{b}_3 = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$, $\mathbf{b}_4 = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$. Find $\mathbf{A}^{-1}$, and use it to solve the four equations $\mathbf{Ax} = \mathbf{b}_1$, $\mathbf{Ax} = \mathbf{b}_2$, $\mathbf{Ax} = \mathbf{b}_3$, $\mathbf{Ax} = \mathbf{b}_4$.

5. (10%) Let the matrix $\mathbf{A} = \begin{bmatrix} 2 & 3 \\ -4 & -5 \end{bmatrix}$. Please find the eigenvalues and the corresponding eigenvectors of **A**.

6. Let A and B be two events associated with an experiment. Suppose that P(A)=0.4 while P($A \cup B$)=0.7. Let P(B)=p.
   (a) (5%) For what choice of p are A and B mutually exclusive?
   (b) (5%) For what choice of p are A and B independent?

7. (10%) Let $G(X) = a\sin(\omega\, t + X)$ where a, ω and t are constants and X is a random variable.

If X has the probability density function (pdf)

$$f_X(x) = \begin{cases} \dfrac{1}{2\pi} & 0 \le x \le 2\pi \\ 0 & otherwise \end{cases},$$

find the mean $E[G(X)]$ and variance Var[$G(X)$]. (Note: $\cos^2\phi = \dfrac{1+\cos 2\phi}{2}$)

8. Let $X_1, X_2, ..., X_n$ be a sample of values from a Gaussian (normal) population having mean μ and variance σ^2. The sample mean is defined by $\overline{X} = \dfrac{1}{n}\sum_{i=1}^{n} X_i$, and the sample variance is defined by $S^2 = \dfrac{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}{n-1}$.

(a) (10%) Show that the moment generating function ($\phi(t) = E[e^{tX}]$) of a Gaussian random variable X with mean μ and variance σ^2 is given by $\phi(t) = \exp\left\{\mu\, t + \dfrac{\sigma^2 t^2}{2}\right\}$.

(b) (10%) Show that $\overline{X}$ is a Gaussian random variable with mean μ and variance $\dfrac{\sigma^2}{n}$.

(Hint: use the result of (a))

(c) (10%) Find $E[S^2]$.