中原大學 100 學年度 **碩士班** 入學考試

3 月 19 日 15:30~17:00　工業與系統工程學系乙組　誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

科目：作業研究　（共 3 頁第 1 頁）

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(1) (10%) A major investment company now has the opportunity to participate in three large construction projects:

Project 1: Construct an office building.

Project 2: Construct a hotel.

Project 3: Construct a shopping center.

Each project requires each partner to pay capital at four different points in time: a down payment now, and additional capital after one, two, and three years.

The following table shows for each project the *total* amount of investment capital required from all partners at four different points in time.

	Investment Capital Requirements		
Year	**Office Building**	**Hotel**	**Shopping Center**
0	$40 million	$80 million	$90 million
1	60 million	80 million	50 million
2	90 million	80 million	20 million
3	10 million	70 million	60 million
Net present value	$45 million	$70 million	$50 million

The company can participate any of the projects in any **percentage**.

The company currently has $25 million available for capital investment. Projections are that another $20 million will become available after one year, $20 million more after two years, and another $15 million after three years. The management wants to invest as much as possible the company's investment capital available. Funds not used at one point time are available at the next point of time. Formulate this problem as the linear programming model as to what percentage share to each project should the company investment so that that the total net present value is maximized?

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(2) (12%) A company must produce a product to meet contracted sales in each of the next four months. The available production and storage facilities are changing month by month, so the production capacities, unit production cost, and unit storage costs vary by month. Therefore, it may be worthwhile to overproduce product in some months and store them until needed.

For each of the four months, the pertinent data is as follows.

Month	Max Production	Unit Cost Of Production	Unit Cost Of Storage	Contracted Sales
1	10	15	1	9
2	8	17	2	5
3	10	19	3	6
4	5	20		7

The production manager wants a schedule developed for the number of units of the product to be produced in each of the four months. The objective is to minimize the total of the production and storage costs while meeting the contracted sales for each month. There is no final inventory is desired after the four months. Formulate this problem as the transportation problem.

(3) The Primal problem.

Maximize $z = 10X_1 + 8X_2$

Subject to

$$2X_1 + X_2 \le 2$$
$$X_1 - 2X_2 \le 1$$
$$X_1 + 3X_2 \le 2$$
$$2X_1 - X_2 \le 4$$

(i) (8%) Solve the primal problem

(ii) (5%) Construct the dual problem for this primal problem.

(iii)(10%) Use the **complementary slackness property** and the optimal solution for the primal problem to find the optimal solution for the dual problem. Show your calculation.

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(4) Markov Chains problem.

Given the following (one-step) transition matrix of a Markov chain:

$P=$

State	0	1
0	**0**	**1**
1	**1**	**0**

(i) (5%) If $X_{10001} = 1$, what is the probability that $X_{10003} = 0$? Explain.

(ii) (3%) Determine the classes of the Markov chain, whether it is open or close? Explain.

(iii)(3%) Determine whether the class is recurrent or transient? Explain.

(iv)(3%) Determine the period of each state?

(v) (4%) The conditional probability $P\{X_{t+1} = j \mid X_t = i\}$ is called one-step transition probability and $P\{X_{t+n} = j \mid X_t = i\}$ is called n-step transition probability. Give the mathematical expressions of the one-step and n-step ***stationary*** transition probabilities.

(vi)(4%) Fill the blank of the mathematical expression of the Markovian property. If $P\{X_{t+1} = j \mid X_0 = k_0, X_1 = k_1, ..., X_{t-1} = k_{t-1}, X_t = i\} = ($　　　　　　　　$)$, for t = 0,1,.., and every sequence $i, j, k_0, k_1, ..., k_{t-1}$.(請作答於答案卷)

(5) (18%) Consider the following nonlinear programming problem.

Max Z= $36X_1 - 6X_1^2 + 20X_2 - 4X_2^2 + 40X_1 - 3X_2^2$

Subject to $X_1 + X_2 + X_3 \leq 3$

All $X_1, X_2, X_3 \geq 0$ and **integers**.

Use Dynamic Programming to solve this problem.

(6) (15%) You are given an M/M/1 queuing system in which the expected waiting time and expected number in the system are 120 minutes and 8 customers, respectively. Determine the probability that a customer's service time exceeds 20 minutes.