題號： 95
科目：應用數學(B)
節次： 2

國立臺灣大學101學年度碩士班招生考試試題

題號： 95
共 / 頁之第 / 頁

**1 (25 points)**

(a) (10 points) Write down the Taylor expansions of $e^x$ and $e^{-x}$. Draw the schematic diagrams of $e^x$ and $e^{-x}$. What are the linear and nonlinear regimes of the $e^x$ and $e^{-x}$.

(b) (10 points) Why is $e^{-1}$ scale used in the science? Why is not $e^{-2}$ or $e^{-3}$ used? What is the $e^{-1}$ scale for the functions of $e^{-k^3 t}$ and $e^{-x^2/n}$?

(c) (5 points) Draw a graph for $f(t) = te^{-t}$ ($t \geq 0$) and explain. What is the maximum of the function and the corresponding $t$.

**2 (15 points)** Find the eigenvalues and eigenvectors (normalized) of the following matrix A and B.

(a) (5 points)

$$A = \begin{pmatrix} 0.8 & 0.6 \\ 0.2 & 0.4 \end{pmatrix}$$

(b) (5 points)

$$B = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$$

(c) (5 points) What are the Sturm-Liouville properties of eigenvectors and eigenvalues for the Hermitian (symmetric) and skew-Hermitian (anti-symmetric) matrices.

**3 (30 points)** Solve the following first order ordianry differential equations with $y_0$ as the initial condition.

(a) (10 points) $dy/dt = -\alpha y$,

(b) (10 points) $dy/dt = 1 - y$,

(c) (10 points) $dy/dt = y - y^2$.

**4 (10 points)** 偵探科南對命案死者死亡時間的推理。科南在命案發生短時間內到達現場, 此時死者溫度仍比室溫爲高, 科南利用

$$\frac{d\triangle T}{dt} = -\alpha \triangle T$$

方程式以及其解

$$\triangle T(t) = \triangle T(t_0) e^{\alpha(t-t_0)}$$

協助推理死亡時間。$\triangle T = T - T_{room}$, 死者溫度和室溫之差。$t_0$是死亡時間 (科南想要知道的答案), $\triangle T(t_0)$爲死者死亡剎那的身體溫度 和室溫的差距, 是已知數。科南需要量幾次死者溫度, 以及如何計算才可以知道死亡時間？

**5 (20 points)** Consider the one dimensional wave equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2},$$

with boundary condition $y(0,t) = y(\pi,t) = 0$, and initial conditions $y(x,0) = f(x)$ and $y_t(x,0) = g(x)$. Solve the equation with the separation variables technique.

Hint: The Fourier sine series of an odd function of period $2\pi$ (and $n$ integers) is

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx,$$

with coefficients

$$b_n = \frac{2}{\pi} \int_0^\pi f(x) \sin nx \, dx.$$

試題隨卷繳回