

中原大學 100 學年度 碩士班 入學考試

3 月 19 日 10:30~12:00

機械工程學系乙組

誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

科目：工程數學

(共 1 頁第 1 頁)

可使用計算機，惟僅限不具可程式及多重記憶者

不可使用計算機

1. Describe the definition and physical meaning of the following terminologies:

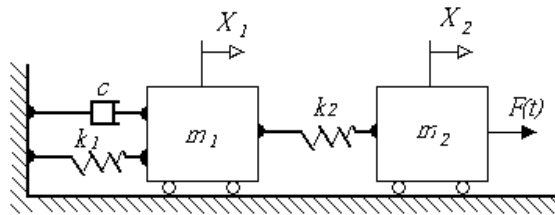
- (a) Laplace Transform (4%)
- (b) Fourier Transform (4%)
- (c) Green's Theorem (4%)
- (d) Stokes's Theorem (4%)
- (e) Conformal Mapping (4%)

2. Find a general solution of the following equations:

- (a) $y' = y - 3x^2$ (6%)
- (b) $y'' - 4y' + 6y = e^{2x} - 3e^{4x}$ (6%)
- (c) $x^2 y'' + 2xy' - 5y = 0$ (6%)
- (d) $y''' - 2y'' + y' - 2y = x^2 - 2x + 4$ (6%)

(e)
$$\begin{cases} 7x_1 - 3x_2 + 4x_3 = -7 \\ 2x_1 + x_2 - x_3 + 4x_4 = 6 \\ x_2 - 3x_4 = -5 \end{cases} \quad (6\%)$$

3. Consider a system in which X_1 and X_2 are displacements from static equilibrium positions and the positive direction is rightward. In case a force $F(t)$ acting on m_2 , (a) find the equations of motion for the system, (b) let $m_1 = 4$, $m_2 = 2$, $k_1 = 2$, $k_2 = 2$, $c = 1$, $F(t) = 1$, solve the equations. (16%)



4. At a point $P(4, -4, 7)$ the stress state referred to axes (x, y, z) is given by $\sigma_{ij} = \begin{bmatrix} 0 & 700 & 0 \\ 700 & 0 & -400 \\ 0 & -400 & 0 \end{bmatrix}$ MPa,

determine (a) the stress vector of the point on the section $2x + 2y - z = -7$, (b) the principal stresses and the maximum shear stress at P . (Hint: the stress vector at a point on a surface is the product of the stress state at the point and the unit normal direction of the surface, the principal stresses are corresponding to the eigen values of the stress state at the point.) (18%)

5. A string of length L is fixed at $x = 0, L$. The governing equation for the string under a forced vibration

is $\frac{\partial^2 y}{\partial x^2} - \frac{1}{c_0^2} \frac{\partial^2 y}{\partial t^2} = -\delta(x - \frac{L}{2})e^{i\omega t}$, where c_0 is a constant and $\delta(x - \frac{L}{2})$ is a delta function.

Assuming that $y(x, t) = Y(x)\exp(i\omega t)$, solve the equation using Fourier transform technique. (16%)