中原大學 100 學年度 **碩士班** 入學考試

3 月 19 日 15:30~17:00　　應用數學系資訊科學組　　誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

科目：離散數學　　（共 2 頁第 1 頁）

□可使用計算機，惟僅限不具可程式及多重記憶者　　■不可使用計算機

請作答於答案卷

一、True or false. (2%×10＝20%)

( ) 1. $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$.

( ) 2. The two compound propositions $\neg P\vee\neg Q\vee\neg R$ and $\neg P\vee(R\rightarrow\neg Q)$ are equivalent.

( ) 3. The "bubble sort" is a sorting that uses passes where successive items are interchanged if they are out of order, and it has $O(n\log n)$ complexity.

( ) 4. A "countable set" is a finite set that can be placed in one-to-one correspondence with the set of positive integers.

( ) 5. The coefficient of x^5y^6 in the expansion of $(x-5y)^{11}$ is $\binom{11}{5}(-5)^6$.

( ) 6. The relation *R* on **Z×Z** defined by *(a,b)R(c,d)* if and only if *(a+d)=(b+c)* is an equivalence relation.

( ) 7. If R_1 and R_2 are reflexive relations on a set *S*, then $R_1\oplus R_2$ is reflexive.

( ) 8. There exits a graph *G* with five nodes (vertices) and the degree sequence being *{5,4,4,3,1}*.

( ) 9. The complete bipartite graph $K_{n,n}$ with $n\geq 3$ is a nonplanar graph.

( ) 10. Let *T* be the minimum spanning tree of a graph *G*. Then the shortest path between any pair of two nodes (vertices) *u* and *v* of *G* is the path between *u* and *v* in *T*.

二、Answer the following questions. (30%)

(4%) 1. State the well-ordering property.

(4%) 2. State the pigeonhole principle.

(4%) 3. List all the derangement of the string 1234.

(4%) 4. State the Four Color Theorem.

(4%) 5. State Dirac's Theorem (for any simple graph to have a Hamiltonian circuit).

(10%) 6. Consider the poset *({3,5,9,15,24,45}, |)*.

6.1) Find the maximal elements.

6.2) Find the minimal elements.

6.3) Is there a greatest element?

6.4) Find all upper bounds of *{3,5}*.

6.5) Find the least upper bound of *{3,5}* if it exists.

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三、Solve the following problems. (12.5%×4=50%)

1. Prove that $1\cdot 2+2\cdot 3+3\cdot 4+...+n(n+1)=n(n+1)(n+2)/3$, where n is a positive integer, by mathematical induction.

2. Consider the equation $x+y+z=11$, where x, y and z are nonnegative integers.
   2.1) How many solutions does it have?
   2.2) How many solutions does it have if the condition $x\geq 1, y\geq 2, z\geq 3$ is satisfied?
   2.3) How many solutions does it have if the condition $5\geq x\geq 2, 6\geq y\geq 3, 7\geq z\geq 4$ is satisfied?

3. Solve the recurrence relation $a_n=6a_{n-1}-11a_{n-2}+6a_{n-3}$ with the initial conditions $a_0=4, a_1=7, a_2=17$.

4. Show that if G is a bipartite simple graph with v nodes (vertices) and e edges, then $e\leq v^2/4$.