# 中原大學100學年度碩士班入學考試

3月19日 13:30 ~ 15:00　　應用數學系統計組

誠實是我們珍視的美德,
我們歡迎 '拒絕作弊, 堅守正直' 的你!

科目: 機率　　共一頁第一頁

不可使用計算機　　請同學依題號順序作答, 並須註明題號。

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1. Let a hand of five cards be drawn at random from an ordinary deck of 52 playing cards. Calculate

   (a) (10pt) the probability of the set of outcomes in which there are exactly two queens, two jacks and one Ace, and

   (b) (10pt) the conditional probability of an all-heart hand, relative to the hypothesis that there are at least 4 hearts in the hand.

2. (10pt) Let the random variable $X$ of the discrete type have the p.d.f. $f(x) = x/10$, $x = 1, 2, 3, 4$, zero elsewhere. Find the distribution finction of $X$ and use it to calculate the probability of the event that $1.5 < X < 3$.

3. (10pt) A median of a distribution of one random variable $X$ is a value of $x$ such that $P(X < x) \leq 1/2$ and $P(X \leq x) \geq 1/2$. If there is only one such $x$, it is called the median of the distribution. Find the median of the binomial(4, 1/4) distribution.

4. Let $X$ have the p.d.f. $f(x) = 4x^3$, $0 < x < 1$, zero elsewhere.

   (a) (10pt) Calculate the mean and variance of $X$.

   (b) (10pt) Find the distribution function and p.d.f. of $Y = -2 \ln X^4$.

5. (10pt) Suppose that the p.d.f. of $X$ and $Y$ is

$$f(x, y) = \begin{cases} 1, & 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

   Let $Z = X + Y$. Find the p.d.f. of $Z$ and $P(Z \leq 1.5)$.

6. (10pt) Suppose that the p.d.f. of $X$ and $Y$ is

$$f(x, y) = \begin{cases} 2, & 0 < x < y < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

   Find the conditional mean and conditional variance of $X$ given $Y = y$.

7. (10pt) Let $X$ and $Y$ have the joint p.d.f. $f(x, y) = 2e^{-x-y}$, $0 < x < y < \infty$, zero elsewhere. Find the joint p.d.f. of $U = 2X$ and $V = Y - X$ and argue that $U$ and $V$ are independent.

8. (10pt) Let $X_1$, $X_2$, $X_3$, $X_4$ be independent and identical distributed random variables from the uniform distribution over the interval $(0, 1)$. Find the p.d.f. of $Y = \min(X_1, X_2, X_3, X_3)$.