

中原大學 100 學年度 碩士班 入學考試

3 月 19 日 10:30~12:00 應用數學系統計組

誠實是我們珍視的美德,

我們喜愛「拒絕作弊, 堅守正直」的你!

科目: 基本數學

(共 1 頁第 1 頁)

可使用計算機, 惟僅限不具可程式及多重記憶者

不可使用計算機

1. (10%) Find $\lim_{x \rightarrow \infty} x^{\frac{3}{2}}(\sqrt{x+2} - 2\sqrt{x+1} + \sqrt{x})$.

2. (10%) Find $\int_0^1 \int_0^{1-y} \frac{1}{1-xy} dx dy$.

3. (10%) Let $F(x) = \int_{x^3}^{x^2} e^{t^2} dt$. Find $F'(x)$.

4. (10%) Find (a) $\int x^2 e^x dx$. (b) $\int_{-2}^2 \frac{\sin \theta}{1 + \cos^2 \theta} + \theta^2 d\theta$.

5. (10%) Find the absolute minimum and maximum values of $f(x, y) = x^2 - 2xy + 2y$ on the $[0, 3] \times [0, 2]$.

6. (20%) Let $A = \begin{pmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{pmatrix}$.

(a) Find the eigenvalues of A .

(b) Find the eigenvectors of A associated with the eigenvalues in (a).

(c) Find A^{100} .

7. (10%) Let A and B be $n \times n$ matrices and $A + B = AB$. Prove that $AB = BA$.

8. (10%) Find $\det \begin{vmatrix} 0 & -1 & 2 & -3 & -6 \\ 1 & 0 & -4 & -5 & 7 \\ -2 & 4 & 0 & 0 & -2 \\ 3 & 5 & 0 & 0 & -5 \\ 6 & -7 & 2 & 5 & 0 \end{vmatrix}$.

9. (10%) Suppose $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and that $T(1, 2) = (2, 4)$, $T(0, 3) = (1, 5)$

(a) Find $T(3, 5)$.

(b) Determine the matrix A of T with respect to base $\beta = \left\{ \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \end{bmatrix} \right\}$.