中原大學 100 學年度 碩士班 入學考試

3 月 19 日 15:30~17:00　　應用數學系數學組 、應用數學系數學組(在職)

誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

科目：線性代數　　（共 1 頁第 1 頁）

□可使用計算機，惟僅限不具可程式及多重記憶者　　■不可使用計算機

1 (20) Let V be the subspace of R^4 spanned by the vectors $v_1 = (1, 2, 2, 1)$, $v_2 = (0, 2, 0, 1)$, $v_3 = (-2, 0, -4, 3)$.

(a) Prove that v_1, v_2, v_3 form a basis for V.

(b) Show that $v = (1, 6, 2, 3)$ lies in V

(c) What are the coordinates of v in (b) relative to the basis (v_1, v_2, v_3) ?

2. (30) Given the matrix

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

(a) What is the rank of A ? (b) Find the inverse of A . (c) What is the characteristic polynomial of A ?

(d) What are its eigenvalues ? (e) Diagonalize A. (f) Explain whether A is positive .

3. (30) Given the following matrices:

$$A = \begin{bmatrix} 2 & i \\ -i & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

(a) Which of the above matrices are skew-symmetric , hermitian, and unitary ?

(b) Is is true that if a matrix M is skew-symmetric , then M^2 is symmetric ? Explain why ?

(c) What kind of matrix preserves norm ?

4. (20) By a direct computation. Show that

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

are not similar .