中原大學 100 學年度 **碩士班** 入學考試

3 月 19 日 13:30~15:00　　應用數學系數學組 、應用數學系數學組(在職)　　誠實是我們珍視的美德，我們喜愛「拒絕作弊，堅守正直」的你！

科目：高等微積分　　（共 1 頁第 1 頁）

□可使用計算機，惟僅限不具可程式及多重記憶者　　■不可使用計算機

1. Let $A = [a, b]$
   $B = [a,b)$
   $C = (a, b)$
   $D = \{(x, y): x^2 + y^2 \le 1\}$
   $E = \{(x, y, z): x^2 + y^2 + z^2 < 1\}$
   Determine which are the compact sets and give reasons to support your answer. (10 points)

2. Prove that S is a closed set $\Leftrightarrow \bar{S} = S$.(10 points)

3. In a metric space M, if subsets satisfy $A \subseteq S \subseteq \bar{A}$, where $\bar{A}$ is the closure of A, then A is said to be *dense* in S. For example, the set **Q** of rational numbers is dense in **R**. If A is dense in S and if S is dense in T, prove that A is dense in T. (10 points)

4. A metric space M is said to be *separable* if there is a *countable* subset A which is dense in M. For example, **R** is separable because the set **Q** of rational number is a countable dense subset. Prove that every Euclidean space $\mathbf{R}^k$ is separable. (10 points)

5. Let S and T be subsets of $\mathbf{R}^n$. Prove that $\overline{S \cap T} \subseteq \bar{S} \cap \bar{T}$ and that $S \cap \bar{T} \subseteq \overline{S \cap T}$ if S is open. (10 points)

6. Prove that in Euclidean space $\mathbf{R}^k$ every Cauchy sequence is convergent. (10 points)

7. Prove that f is continuous on S if, and only if,

   $f(\bar{A}) \subseteq \overline{f(A)}$ for every subset A of S.(10 points)

8. Give an example of a continuous function $f: S \to T$ and a Cauchy sequence $\{x_n\}$ in metric space S for which $\{f(x_n)\}$ is not a Cauchy sequence in metric space T. (10 points)

9. Assume that f is uniformly continuous on a bounded set S in $\mathbf{R}^n$. Prove that f must be bounded on S. (10 points)

10. Assume that $f_n \to f$ uniformly on S. If each f_n is continuous at a point c of S, prove the limit function f is also continuous at c. (10 points)