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1. Answer the followings in probability theory: (20%)

(a) Define the 'random variable'. (5%)

(b) Please describe the relation 'P[A|B]=P[A]' (A and B are independent) by using a Venn diagram in which the event areas are proportional to their probabilities. (5%)

(c) For a random variable Y and two different values a and b from the range of Y, please explain why the two events, $\{Y=a\}$ and $\{Y=b\}$, are disjoint? (5%)

(d) Is it possible that two uncorrelated random variables A and B are also orthogonal? (5%)

2. We redefine a new cumulative distribution function of a random variable X as $H_X(x)=p[X<x]$. In such definition, let $Y=g(X)$ where

$$g(x)=\begin{cases}-1\,, & x\le -2\\ 0\,, & -2<x<2\\ 1\,, & x\ge 2\end{cases}$$

Please express $H_Y(y)$ in terms of $H_X(x)$. (15%)

3. X is a Gaussian (1, 2) random variable. Let a derived random variable $Y=\sqrt{X}$. Could we find the second moment of Y directly from the probability density function of X? (15%)

4. Decode the messages of base station. (9%)

(a) Base station sends messages to relay station using the coding matrix $\mathbf{A}=\begin{bmatrix}6 & -5\\ -1 & 1\end{bmatrix}$. The relay station sends messages to user station using the coding matrix $\mathbf{B}=\begin{bmatrix}1 & 2\\ 1 & 3\end{bmatrix}$. Find the encoding matrix that enables base station to send messages directly to user station. (3%)

(b) Using the encoding matrix in (a), if the user station receives the messages 49, 38, -5, -3, -61, -39, please help user station to decode the messages of base station. (6%)

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5. Please solve the following equations. (10%)

(a) Determine the equation of the polynomial of degree two whose graph passes through the points (-1,-1), (0,1), (1,-3). (5%)

(b) Find the equation of the least square line that best fits the points (1,1), (2,5), (3,9). (5%)

6. Consider the linear operator T on P_1. (15%)

$$T(ax+b)=(a-2b)x+(-2a+b)$$

(a) Find the matrix representation **A** of linear operator T with respect to standard basis $B=\{x,1\}$ of P_1. (3%)

(b) Use a similar transformation to determine the matrix $\mathbf{A}'$ of T with respect to basis $B'=\{x+1,2x+1\}$ of P_1. (5%)

(c) Use a similar transformation to find the diagonal matrix $\mathbf{A}''$ representation of T and determine the basis B'' of P_1 for this representation. (7%)

7. Solve the difference equation. (8%)

$$a_n=a_{n-1}+2a_{n-2}, \qquad \text{for } n=3,4,5,\cdots$$

with initial conditions $a_1=1$ and $a_2=3$. Use the initial conditions to determine a_{15}.

8. State (with a brief explanation) whether the following statements are true or false. (8%)

(a) The set M_{22} of real 2×2 matrices is a vector space. The subset of symmetric matrices of the form $\begin{bmatrix} a & b \\ b & a^2 \end{bmatrix}$ is a subspace of M_{22}, where a and b are real numbers. (2%)

(b) There exists a set that is linear independent but does not span the real vector space $\mathbf{R}^3$. (2%)

(c) Let $T: U\to V$ be a linear mapping. Let **v** be a nonzero vector in V. Let W be the set of vectors **w** in the domain U such that $T(\mathbf{w})=\mathbf{v}$. Therefore, W is a subset of U. (2%)

(d) $\{(1,0,1),(2,1,5)\}$ is a basis for the subspace of vectors in $\mathbf{R}^3$ of the form $(a, b, a+3b)$, where a and b are real numbers. (2%)