科目	微積分	適用系所	統計學系統計與精算碩士班應用統計暨計量財務組、精算組	時間	90分鐘

1) (10%) Let $h(z) = ze^{-z^2}$, find $h^{(n)}(0)$, the nth derivative of h at $z = 0$.

2) (15%) Given the cost function $C(x) = 0.02x^3 - 0.2x^2 + x + 1$ and the price function

   $P(x) = 1000 - 0.01x$.

   (a) Define the average cost function by $A(x) = C(x)/x$.

   Find x such that $A(x)$ is minimized. (7%)

   (b) Define the profit function by $\pi(x) = R(x) - C(x)$ where $R(x) = xP(x)$ is the revenue

   function. Find x such that $\pi(x)$ is maximized. (8%)

3) (10%) Find the maximum and the minimum (if any) of the function

   $$f(x, y) = -2x^2 - 3y^2 + 3xy - 7x + 4y$$

4) (15%) Find the maximum and the minimum (if any) of the function

   $f(x_1, x_2) = -x_1^2 - x_2^2 + 4x_1 + 6x_2$ under the constraints $x_1 + x_2^2 \leq 1$.

5) (17%) Evaluate the integral

   (a) $\int_2^3 \left(x^2 + 2x + \frac{1}{3}\right) dx$ (10%)

   (b) $\int_{1.5}^4 \sqrt{2x + 1}\, dx$ (7%)

6) (10%) The function $f(x)$ is given by

   $$f(x) = \begin{cases} C(4x - 2x^2), & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

   where C is a constant. Suppose that $\int_{-\infty}^{\infty} f(x)\, dx = 1$. Find the value of C.

7) (15%) Evaluate the double integral

   (a) $\int_0^{\ln 2} \int_0^{\ln 5} e^{2x-y}\, dx\, dy$ (7%)

   (b) $\iint_R 10x^2 y^2\, dxdy$, where $R = \{(x, y) | 0 \leq y \leq x \leq 1\}$ (8%)

8) (8%) $f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$, where $\lambda > 0$ is a constant. Evaluate $\int_0^{\infty} x^2 f(x)\, dx$.