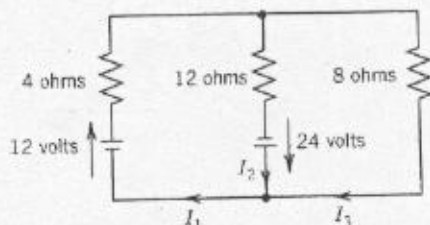● 不可使用電子計算機

1. Using Variation of Parameters to find a solution of the following equation **(16%)**

$$x^2 y'' - 3x y' + 4y = \ln x \quad , \quad x > 0$$

2. Using the method of Laplace Transformation to solve the initial value problem of $y(t)$

   (17%)

$$y'' + 4y' + 13y = 2 e^{-2t} \sin 3t \quad \text{with} \quad y(0) = 1 \quad , \quad \left.\frac{dy}{dt}\right|_{t=0} = 0$$

3. Using Kirchhoff's laws, find the currents in the following networks. **(10 %)**

4. Find out what type of conic section (or pair of straight lines) is represented by the given

   quadratic form $4x_1 x_2 + 3x_2^2 = 1$. Transform it to principal axes. Express $\vec{x}^T = [x_1 \ x_2]$ in

   terms of the new coordinate vector $\vec{y}^T = [y_1 \ y_2]$. **(10 %)**

5. Evaluate $\int_C \vec{F} \bullet \vec{r}' ds$, $\vec{F} = (x^2 + y^2)^{-1}[-y, x, 0]$, $C : x^2 + y^2 = 1$, $z = 0$, oriented

   clockwise. **(13 %)**

100070

●不可使用電子計算機

6. Use Laplace transformation to solve the P.D.E. problem

$$\frac{\partial w}{\partial x} + x\frac{\partial w}{\partial t} = 0, \quad w(x,0) = 0, w(0,t) = t$$

w(x, t) = ?　　　(17%)

7. There is periodic square wave with analytic represented as f(x) function

$$f(x) = \begin{cases} -k \text{ when } & -\pi < x < 0 \\ k \text{ when } & 0 < x < \pi \end{cases} \quad \text{and } f(x+2\pi) = f(x)$$

Please find the Fourier coefficient of a_n, b_n and their series functions to present the f(x) functions.　　(17%)

100071