元智大學 100 學年度研究所 **碩士班** 招生試題卷

系(所)別：生物與醫學資訊碩士學位學程　組別：不分組　科目：離散數學　

●不可使用電子計算機

Notation:

- $\mathbb{Z}$　The set of integers
- $\mathbb{Z}^+$　The set of positive integers
- $\mathbb{N}$　The set of natural numbers (including 0)

一、填充題（每格 5 分，共 70 分）

1. For $x \in \mathbb{Z}^+$, let $F(x) = 3x! + x^2$ and $G(x) = 6x^3 + 10x$.
   - A. A tight (as good as possible) upper (**big-O**) bound of $F(x)$ is _______.
   - B. A tight (as good as possible) lower (**big-Ω**) bound of $(G + F)(x)$ is _______.

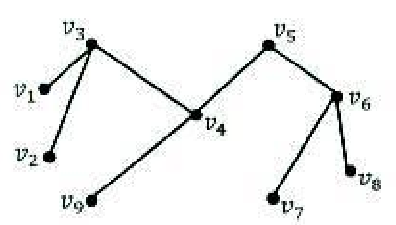
2. Let $F: \mathbb{N} \to \mathbb{N}$, $F(x) = x! - x$.
   - A. Is $F(x)$ an **one-to-one** function? _______ (true or false)
   - B. Is $F(x)$ an **onto** function? _______ (true or false)

3. Let $S = \{x \in \mathbb{Z}^+ \mid x < 8\}$.
   - A. The number of **4-permutations** of S **containing 41** is _______.
   - B. The number of **4-combinations** of S **containing 41** is _______.

4. For a **full n-ary** tree T of **height h**,
   - A. The **maximum** number of **leaf** vertices in T is _______.
   - B. The **minimum** number of **internal** vertices in T is _______.

5. For a binary relation $R = \{ (s_1, s_2) \mid s_1, s_2 \in \mathbb{Z}, |s_1| \neq |s_2| \}$,
   - A. Is R a **reflexive** binary relation? _______ (true or false)
   - B. Is R a **symmetric** binary relation? _______ (true or false)

6. Let v_5 be the **root** of the following **ordered** rooted tree T:
   [Note: Vertices are ordered as the values of their subscripts]



   - A. The **sub-tree** of T, with v_4 as its root, is _______.
   - B. For T, the vertex visiting sequence of the **breadth-first** traversal is _______.

7. Let $S = \{x \in \mathbb{Z}^+ \mid x \leq 16, x \neq 3n, n \in \mathbb{Z}^+\}$ and $T = \{x \in \mathbb{Z}^+ \mid x \leq 16, x = 2n, n \in \mathbb{Z}^+\}$.
   - A. The number of elements in $S \times T$ is _______.
   - B. The number of different **binary relations** from S to T is _______.

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二、問答題（每題 10 分，共 30 分）

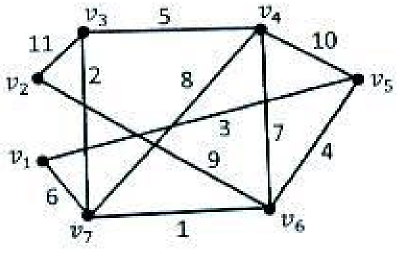
1. Let p, q, and r denote propositions. Prove that $(p \to q) \to (r \to \neg q)$ and $\neg(r \wedge q)$ are logically equivalent **using logical equivalence laws**.
   [Hint: For two propositions m and n, $m \to n \equiv \neg m \vee n$, and $m \vee (m \wedge n) \equiv m$.]

2. For $a, b, c, d \in \mathbb{Z}^+$, prove that if $c = \mathbf{lcm}(a, b)$ and $d = \mathbf{gcd}(a, b)$, then d^3 **divides** $(a \cdot b \cdot c)$.
   [Note: $\mathrm{lcm}(\cdot,\cdot)$ and $\mathrm{gcd}(\cdot,\cdot)$ denote the **least common multiple** and the **greatest common divisor** of two positive integers, respectively.]

3. For a weighted (undirected) graph $G = (V, E)$, the **Kruskal's algorithm** can be applied to find the **minimum spanning tree** $T = (V', E')$ of G. Following is the pseudo-code of the Kruskal's algorithm:

```
Procedure Kruskal(G) : G = (V,E), V = {v1, v2, ..., vn}
begin
  V' = ∅, E' = ∅, T = (V',E')
  for i = 1 to n − 1
    Select ei = {vj, vk} ∈ E (for ei ∉ E') with minimal weight without forming
    simple circling paths in T
    V' = V' ∪ {vj, vk}, E' = E' ∪ {ei}
  end for
  return T
end
```

By using the Kruskal's algorithm, obtain the minimum spanning tree of the following weighted (undirected) graph:



[Note: Show the partial result step by step, i.e. draw the partial spanning tree **at the end of each *for* loop**.]

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